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**On Decomposing Net Final Values: Systemic Value  
Added and Shadow Project**

by

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# ON DECOMPOSING NET FINAL VALUES: SYSTEMIC VALUE ADDED AND SHADOW PROJECT

**ABSTRACT.** A new decomposition index is proposed for capital budgeting purposes, based on a systemic approach. Relations with other decomposition models are studied, among which Stewart's (1991). The index here introduced differs from Stewart's EVA in that we do not need capitalize cash flows to obtain a project's Net Final (or Present) Value. It rests on a different interpretation of the notion of residual income and is formally connected with the EVA model by means of a *shadow* project, which enables us to regard the periodic Systemic Value Added as an Economic Value Added. Some results are offered, providing sufficient and necessary conditions for decomposing Net Final Values. Relations between project  $P$ 's EVA and *shadow* project's EVA are studied and as a nice by-product we are left with an index that is capable of integrating accounting and financial calculus in appraising investments.

**Keywords:** decomposition, residual income, systemic, shadow project, EVA.

## Introduction

The problem of decomposing a cash flow stream has gained in recent years a renewed interest in both american and continental literature. I especially shall dwell on the contributions of Stewart (1991), Peccati (1987, 1992), Pressacco and Stucchi (1997). Stewart proposes the Economic Value Added, which formally translates the economic concept of residual income. Peccati decomposes the Net Present Value of a project, and Pressacco and Stucchi generalize Peccati's model in the sense of Teichroew, Robichek and Montalbano (1965a, 1965b), by introducing a two-valued rate for the project balance. After briefly showing that the three decomposition models bear a strong resemblance one another from a formal point of view, a different decomposition model is proposed, based on a different notion of residual income. I name the index here presented Systemic Value Added (henceforth, often SVA). The relations this model bears to the other ones are investigated thoroughly: all results obtained by Peccati and by Pressacco and Stucchi can be integrated in the SVA model. Further, their Theorems can be proved by resting on the SVA model and the relations between Systemic Value Added and Economic Value Added are pointed out. The SVA model can also be interpreted as an EVA model, where the Economic Value Added is not referred to the project at hand, but to its *shadow project*, whose introduction is significant economically as well as from a formal point of view.

## 1. Stewart's model

The basic objective of EVA is to create a measure of periodic performance based on the concept of residual income.<sup>1</sup> Let  $TC$  be the total capital invested in the project at the outset of period  $s$ ; to

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<sup>1</sup>The EVA is used for projects as well as for firms, in order to compute the value of the firm, or as a tool for rewarding managers (see Biddle, Bowen and Wallace (1999) and O'Byrne (1999)).

compute the EVA, Stewart suggests us to calculate the project's (or firm's) total cost of capital, given by the product of the Weighted Cost of Capital (WACC) and the total capital invested (TC). Then the total cost of capital is subtracted from the Net Operating Profit After Taxes (NOPAT). Notationally, we have, for period  $s$ ,

$$\text{EVA}_s = \text{NOPAT} - \text{WACC} * \text{TC}. \quad (1)$$

### Peccati's model

Consider a project  $P$  whose initial outlay is  $-a_0 < 0$ , with subsequent periodic cash flows  $a_s \in \mathbb{R}$  at time  $s=1, 2, \dots, n$ . Suppose that the evaluator currently invests her wealth in an asset  $C$  whose rate of return is  $i$ . She is faced with the alternative of

- (i) withdrawing the sum  $a_0$  from asset  $C$  and investing it in project  $P$ , or
- (ii) keeping the sum invested at the rate  $i$ .

Then, the rate  $i$  is the so-called opportunity cost of capital. Let  $E_0$  be the initial net worth,<sup>2</sup>  $E_0 \in \mathbb{R}$ . The Net Final Value (NFV) of project  $P$  is given by the difference between alternative final net worths. Denote with  $E_n$  and  $E^n$  the evaluator's net worth at time  $n$ , relative to case (i) and case (ii) respectively. We have

$$\begin{aligned} \text{NFV}(i) &= E_n - E^n \\ &= (E_0 - a_0)(1+i)^n + \sum_{s=1}^n a_s(1+i)^{n-s} - E_0(1+i)^n \\ &= -a_0(1+i)^n + \sum_{s=1}^n a_s(1+i)^{n-s}. \end{aligned} \quad (2)$$

(2) presupposes that  $C$  is an account where the cash flows released by project  $P$  are reinvested in (if positive) or withdrawn from (if negative). The Net Present Value (NPV) is

$$\text{NPV} = \frac{\text{NFV}}{(1+i)^n} = -a_0 + \sum_{s=1}^n a_s(1+i)^{-s}.$$

Assume  $x$  is an internal rate of return for  $P$ . The *outstanding capital* or *project balance*  $w_s$  at the rate  $x$  is defined as

$$\begin{aligned} w_0 &:= a_0 \\ w_s &:= w_{s-1}(1+x) - a_s \quad s = 1, \dots, n. \end{aligned}$$

We have, obviously,

$$w_n = \text{NFV}(x) = 0.$$

To decompose the NPV (NFV) of project  $P$ , Peccati uses the following argument: At the outset of each period  $s$  the investor invests in a (fictitious) uniperiodic project, whose initial outlay is  $-w_{s-1}$ . At the

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<sup>2</sup>The term "net worth" is to be intended as a synonym of wealth.

end of the period, she will receive the sum  $a_s$  along with the value  $w_s$ . Denoting with  $g_s$  ( $G_s$ ) the Net Present Value (Net Final Value) of this uniperiodic project we have

$$g_s = \frac{-w_{s-1}}{(1+i)^{s-1}} + \frac{w_s + a_s}{(1+i)^s} = \frac{w_{s-1}(x-i)}{(1+i)^s} \quad (3a)$$

and

$$G_s = g_s(1+i)^n = w_{s-1}(x-i)(1+i)^{n-s}. \quad (3b)$$

$g_s$  and  $G_s$  are then the quota of the project's NPV (NFV) generated in period  $s$ . Using the project balance equation, it is easy to verify that summing for  $s$  we have

$$\sum_{s=1}^n g_s = \text{NPV}$$

$$\sum_{s=1}^n G_s = \text{NFV}$$

Peccati then extends its model and assumes that the investment is partly financed by a loan contract consisting of an initial receipt  $f_0 > 0$  and subsequent cash flows  $f_s \in \mathbb{R}$  at time  $s=1, \dots, n$ . The *outstanding debt* or *debt balance* at the debt rate  $\delta$  is defined as

$$\begin{aligned} D_0 &:= f_0 \\ D_s &:= D_{s-1}(1+\delta) - f_s \quad s = 1, \dots, n. \end{aligned}$$

Using the same argument as before, modified so as to take debt into account, we have

$$g_s = \frac{-w_{s-1} + D_{s-1}}{(1+i)^{s-1}} + \frac{w_s + a_s + D_s - f_s}{(1+i)^s} = \frac{w_{s-1}(x-i) - D_{s-1}(\delta-i)}{(1+i)^s} \quad (4a)$$

and

$$G_s = g_s(1+i)^n = (w_{s-1}(x-i) - D_{s-1}(\delta-i))(1+i)^n. \quad (4b)$$

Summing for  $s$  we have the NPV and the NFV respectively.

### 3. Pressacco and Stucchi's model

Pressacco and Stucchi (henceforth P&S) extend the first version of Peccati's model by allowin for two pairs of rates  $(i_P, i_N)$  and  $(x_P, x_N)$  in the sense we now show.

The balance of asset  $C$ , denoted by  $C_s$ , is defined as

$$\begin{aligned} C_0 &:= -a_0 \\ C_s &:= C_{s-1}(1+i(C_{s-1})) + a_s \quad s = 1, \dots, n \end{aligned} \quad (5a)$$

with

$$\begin{aligned} i(C_{s-1}) &= i_P && \text{if } C_{s-1} > 0, \\ i(C_{s-1}) &= i_N && \text{if } C_{s-1} < 0, \end{aligned}$$

with  $i_P \neq i_N$  ( $P$  stands for “positive”,  $N$  for “negative”),<sup>3</sup> and an internal pair  $(x_P, x_N)$  is introduced so that

$$\begin{aligned} w_0 &:= a_0 \\ w_s &:= w_{s-1}(1 + x(w_{s-1})) - a_s \quad s = 1, \dots, n \end{aligned} \quad (5b)$$

with

$$\begin{aligned} x(w_{s-1}) &= x_P && \text{if } w_{s-1} > 0, \\ x(w_{s-1}) &= x_N && \text{if } w_{s-1} < 0 \end{aligned}$$

so that  $w_n=0$ .<sup>4</sup>

Therefore, P&S generalize Peccati’s model only under a particular perspective. In fact, they assume  $D_s=0$  for all  $s$  whereas Peccati allow for  $D_s \neq 0$ ; conversely, they handle reinvestment and external financing by introducing the pair  $(i_P, i_N)$  where  $i_N$  acts just whenever the value of  $C$  is negative (Peccati’s model can be seen as assuming  $i_P=i_N=i$ ).<sup>5</sup>

As one can note, the assumption  $C_0=-a_0$  is equivalent to the assumption  $E_0=0$  in Peccati’s model, and the entire model is tied to this assumption.<sup>6</sup> The project’s NFV is then

$$\text{NFV} = E_n - E^0 = -a_0(1 + i(C))^{0,n} + \sum_{s=1}^n a_s(1 + i(C))^{s,n} \quad (6)$$

where

$$\begin{aligned} (1 + i(C))^{s,n} &:= \prod_{k=s+1}^n (1 + i(C_{k-1})) && s < n \\ (1 + i(C))^{s,n} &:= 0 && s = n \end{aligned}$$

The main result of P&S can be summarized as follows:

**P&S Theorem.** *Assume  $C_0=-a_0$ . Peccati’s model can be generalized in a two-rate capitalization of periodic shares so that*

$$G_s = w_{s-1}(x_P - i_N)(1 + i(C))^{s,n}$$

<sup>3</sup>In this paper the notational conventions and the presentation of P&S’s model differ considerably from P&S’s exposition. Our exposition is consistent with the systemic outlook we shall develop later.

<sup>4</sup>I shall never define the value of a rate when its argument is zero, so we can pick whatever value according to our needs.

<sup>5</sup>P&S take as a starting point the idea of Teichroew, Robichek and Montalbano (henceforth TRM) of a project balance depending on two rates. Notwithstanding, TRM rest on the Net Present Value rule, as they assume that unlimited funds are available to the investor and can be employed by the investor at the same rate  $\varrho$ : with our notations, this means  $i_P=i_N=\varrho$ , so that account  $C$  evolves according to the recurrence equation

$$C_s = C_{s-1}(1 + \varrho) + a_s.$$

P&S’s treatment is such that they do not merely allow for an internal pair  $(x_P, x_N)$ , but generalize further on and introduce an external pair  $(i_P, i_N)$ . Under these assumptions, the NPV rule cannot be applied any more and the choice between two or more alternative courses of action must be based on the net final values.

<sup>6</sup>From now on, we will use the two assumptions interchangeably.

or

$$G_s = w_{s-1}(x_N - i_P)(1 + i(C))^{s,n}$$

if and only if

$$x(w_{s-1}) = x_P \quad \text{iff} \quad i(C_{s-1}) = i_N.$$

In such a case, we have

$$\text{NFV} = \sum_{s:w_{s-1}>0}^n w_{s-1}(x_P - i_N)(1 + i(C))^{s,n} + \sum_{s:w_{s-1}<0}^n w_{s-1}(x_N - i_P)(1 + i(C))^{s,n}.$$

#### 4. Relations among the three models

Stewart's EVA model and Peccati's decomposition model are akin: the numerator of  $g_s$  is just the Economic Value Added. In fact (1) can be rewritten as

$$\text{EVA}_s = \text{ROA} * \text{TC} - \frac{(\text{ROD} * \text{Debt} + i * \text{Equity})}{\text{Debt} + \text{Equity}} * \text{TC} \quad (7a)$$

whence

$$\begin{aligned} \text{EVA}_s &= \text{ROA} * \text{TC} - \text{ROD} * \text{Debt} - i * (\text{TC} - \text{Debt}) \\ &= \text{TC} * (\text{ROA} - i) + \text{Debt} * (i - \text{ROD}) \end{aligned} \quad (7b)$$

where ROA is the Return on Assets, ROD is the Return on Debt, and  $i$  is the opportunity cost of capital. All values in (7) obviously refer to period  $s$ . Computing the Economic Value Added for project  $P$ , we have  $\text{TC} = w_{s-1}$ ,  $\text{ROA} = x$ ,  $\text{Debt} = D_{s-1}$ ,  $\text{ROD} = \delta$ , so that

$$w_{s-1}(x - i) + D_{s-1}(i - \delta) = \text{EVA}_s.$$

The relation between (4) and (1) is then given by

$$\begin{aligned} g_s &= (w_{s-1}(x - i) - D_{s-1}(\delta - i))(1 + i)^{-s} \\ &= \text{EVA}_s(1 + i)^{-s} \end{aligned} \quad (8a)$$

and

$$\begin{aligned} G_s &= (w_{s-1}(x - i) - D_{s-1}(\delta - i))(1 + i)^{n-s} \\ &= \text{EVA}_s(1 + i)^{n-s}. \end{aligned} \quad (8b)$$

Consequently, P&S's model can be viewed as a formal extension of Stewart's model in the same sense it generalizes Peccati's model:  $w_{s-1}(x_P - i_N)$  and  $w_{s-1}(x_N - i_P)$  are the numerators of  $g_s$  in the case  $x(w_{s-1}) = x_P$ ,  $i(C_{s-1}) = i_N$ , and in the case  $x(w_{s-1}) = x_N$ ,  $i(C_{s-1}) = i_P$  respectively.

#### 5. The Systemic Value Added

In this section we propose a different decomposition model, based on the notion of *system*. The investor's net worth is seen as a financial system structured in various accounts, which are periodically

activated to consider withdrawals and reinvestments of cash flows. We assume, like P&S, that the balances are functions of a two-valued rate, but we generalize allowing for whatever  $E_0$ . The financial system presents a different structure according to the alternative selected. We can depict it by means of a double-entry sheet where sources and uses of funds are pointed out. If alternative (i) is followed then we have, at time  $s$ ,

$$\begin{array}{c|c}
 \underline{\text{Uses}} & \underline{\text{Sources}} \\
 C_s & E_s \\
 w_s & \\
 \hline
 & 
 \end{array} \tag{9a}$$

for  $s, s=0,1,\dots,n$ , where  $C_s, w_s$  are the balances for asset  $C$  and project  $P$  respectively, and  $E_s$  is the investor's wealth (which, we remind, is allowed to be zero or negative). The structure evolves diachronically according to the recurrence equations (5a) (where the initial condition is replaced by the more general  $C_0=E_0-a_0, E_0 \in \mathbb{R}$ ), (5b), and (5c) here added:

$$E_s = C_s + w_s = E_{s-1} + i(C_{s-1})C_{s-1} + x(w_{s-1})w_{s-1}. \tag{5c}$$

If alternative (ii) is instead selected, we have, at time  $s$ ,

$$\begin{array}{c|c}
 \underline{\text{Uses}} & \underline{\text{Sources}} \\
 C^s & E^s \\
 \hline
 & 
 \end{array} \tag{9b}$$

for  $s, s=0,1,\dots,n$  where  $C^s$  and  $E^s$  denote the values of asset  $C$  and net worth respectively. The financial system is then de-structured, so to say, and  $C^s$  coincides with  $E^s$  for all  $s$ . The rate of interest for account  $C$  will be obviously  $i_P$  or  $i_N$  depending on the sign of  $C^s$ . We describe these facts with the recurrence equation governing the evolution of the system:

$$\begin{aligned}
 E^0 &= C^0 = E_0 \\
 E^s &= C^s = C^{s-1}(1 + i(C^{s-1})) = E^{s-1}(1 + i(E^{s-1}))
 \end{aligned} \tag{10}$$

with

$$\begin{aligned}
 i(C^{s-1}) &= i_P && \text{if } C^{s-1} > 0, \\
 i(C^{s-1}) &= i_N && \text{if } C^{s-1} < 0.
 \end{aligned}$$

Thanks to (10), we can also write

$$i(C^{s-1}) = i(E_0) \quad \text{for all } s \geq 1.$$

Under this systemic perspective, the residual income for period  $s$  is given by the difference between what the investor would earn in that period if she chooses alternative (i) and what she would earn



should she decide to keep on investing at the rate  $i$ , i.e. alternative (ii). This is formally translated in the difference between net profits relative to the two courses of action. The net profit *sub* (i) is

$$E_s - E_{s-1} = i(C_{s-1})C_{s-1} + x(w_{s-1})w_{s-1}, \quad (11a)$$

whereas for (ii) we have

$$E^s - E^{s-1} = i(C^{s-1})C^{s-1}. \quad (11b)$$

(11a) informs us that if the investor undertakes project  $P$  her profit will be given by the return on the capital invested in the project (equal to  $x(w_{s-1})w_{s-1}$ ) added to the interest gained on asset  $C$  (equal to  $i(C_{s-1})C_{s-1}$ ). (11b) informs us that the net profit for (ii) is just the return on asset  $C$  (equal to  $i(C^{s-1})C^{s-1}$ ). The residual income for each period  $s$ , here named *periodic Systemic Value Added* ( $SVA_s$ ), is then

$$\begin{aligned} SVA_s &= (E_s - E_{s-1}) - (E^s - E^{s-1}) \\ &= x(w_{s-1})w_{s-1} + i(C_{s-1})C_{s-1} - i(C^{s-1})C^{s-1}. \end{aligned} \quad (12)$$

Summing for  $s$  we have the (overall) *Systemic Value Added* of project  $P$ . The latter coincides with the Net Final Value of  $P$ :

$$SVA = \sum_{s=1}^n SVA_s = \sum_{s=1}^n (E_s - E_{s-1}) - (E^s - E^{s-1}) = E_n - E^n = NFV. \quad (13)$$

Further, we have

$$\begin{aligned} SVA &= NFV \\ &= E_n - E^n \\ &= E_0 \left( (1 + i(C))^{0,n} - (1 + i(E_0))^n \right) - a_0 (1 + i(C))^{0,n} + \sum_{s=1}^n a_s (1 + i(C))^{s,n} \end{aligned} \quad (14)$$

since

$$\begin{aligned} E_n &= (E_0 - a_0)(1 + i(C))^{0,n} + \sum_{s=1}^n a_s (1 + i(C))^{s,n} \\ E^n &= E_0(1 + i(C^0))^n = E_0(1 + i(E_0))^n \end{aligned}$$

Note that picking  $E_0=0$  (i.e.  $C_0=-a_0$ ) we get to (6) as in P&S's model.

## 6. EVA, SVA and $\overline{EVA}$

**Definition 1:** A pair  $(i_P, i_N)$  is said to be a twin-pair if for all  $s$ ,  $i(C^s)=i(C_s)$

**Definition 2:** A pair  $(i_P, i_N)$  is said to be an  $i_P$ -twin-pair if it is a twin-pair and  $i(C_s)=i_P$ . A pair  $(i_P, i_N)$  is said to be an  $i_N$ -twin-pair if it is a twin-pair and  $i(C_s)=i_N$ .

**Definition 3:** A project  $\overline{P}$  is said to be the *shadow project* of  $P$  (or the *shadow* of  $P$ ) if it consists of the sequence of cash flows

$$(-\overline{a}_0, \overline{a}_1, \dots, \overline{a}_n)$$

available at time  $0, 1, \dots, n$  respectively, such that

$$\begin{aligned}\bar{a}_0 &= a_0 \\ \bar{a}_s &= a_s + \text{SVA}_s \quad s = 1, 2, \dots, n.\end{aligned}$$

Let us have the following notations:

$$\bar{w}_s := C^s - C_s$$

and

$$\begin{aligned}\bar{x}(\bar{w}_{s-1}) &:= \bar{x}_P && \text{if } \bar{w}_{s-1} > 0 \\ \bar{x}(\bar{w}_{s-1}) &:= \bar{x}_N && \text{if } \bar{w}_{s-1} < 0\end{aligned}$$

where

$$\bar{x}_P := x_P \frac{w_{s-1}}{w_{s-1}} \quad \text{and} \quad \bar{x}_N := x_N \frac{w_{s-1}}{w_{s-1}}.$$

Then we have the following

**Definition 4:** The shadow pair  $(\bar{x}_P, \bar{x}_N)$  and the internal pair  $(x_P, x_N)$  are said to be parallel if, for all  $s$ ,

$$x(w_{s-1}) = x_P \quad \text{iff} \quad \bar{x}(\bar{w}_{s-1}) = \bar{x}_P.$$

For the sake of convenience we shall label some propositions occurring frequently in the paper with the following notations:

(Par):= the internal pair  $(x_P, x_N)$  and the shadow pair  $(\bar{x}_P, \bar{x}_N)$  are parallel

(SP):=  $P$  is a Soper project

( $\overline{SP}$ ):=  $\overline{P}$  is a Soper project

(Twin):=  $(i_P, i_N)$  is a twin-pair

( $i_P$ -Twin):=  $(i_P, i_N)$  is an  $i_P$ -twin-pair

( $i_N$ -Twin):=  $(i_P, i_N)$  is an  $i_N$ -twin-pair

In the sequel, we shall assume  $x_P \neq x_N$  and,  $i_P \neq i_N$  unless otherwise specified.

**Lemma 6.1.** *We have*

$$\bar{w}_s = \bar{w}_{s-1}(1 + i(C_{s-1})) - a_s \quad s = 1, \dots, n \quad (\otimes)$$

if and only if (Twin).

*Proof:* Assume (Twin) We have

$$\begin{aligned}\bar{w}_s &= C^s - C_s \\ &= C^{s-1}(1 + i(C^{s-1})) - (C_{s-1}(1 + i(C_{s-1})) + a_s) \\ &= [\text{for (Twin)}] = (C^{s-1} - C_{s-1})(1 + i(C_{s-1})) - a_s \\ &= \bar{w}_{s-1}(1 + i(C_{s-1})) - a_s.\end{aligned}$$

Assume now  $(\otimes)$ . We have

$$\begin{aligned}
C^{s-1}(1 + i(C^{s-1})) - (C_{s-1}(1 + i(C_{s-1})) + a_s) &= C^s - C_s \\
&= \bar{w}_s \\
&= [\text{for } \otimes] = \bar{w}_{s-1}(1 + i(C_{s-1})) - a_s \\
&= (C^{s-1} - C_{s-1})(1 + i(C_{s-1})) - a_s
\end{aligned}$$

whence

$$i(C^{s-1})C^{s-1} - i(C_{s-1})C_{s-1} = i(C_{s-1})(C^{s-1} - C_{s-1})$$

which implies (Twin).

(Q.E.D.)

Lemma 6.1 implies that if (Twin), then

$$\bar{w}_s = a_0(1 + i(C))^{0,s} - \sum_{k=1}^s a_k(1 + i(C))^{k,s} \quad s = 1, \dots, n.$$

Also, (5b) implies

$$w_s = a_0(1 + x(w))^{0,s} - \sum_{k=1}^s a_k(1 + x(w))^{k,s} \quad s = 1, \dots, n,$$

where

$$\begin{aligned}
(1 + x(w))^{k,s} &:= \prod_{h=k+1}^s (1 + x(w_{h-1})) && k < s \\
(1 + x(w))^{k,s} &:= 0 && k = s.
\end{aligned}$$

Then  $\bar{w}_s$  is just  $w_s$  where we substitute  $i(C_{s-1})$  for  $x(w_{s-1})$ .

**Lemma 6.2.** *We have*

$$\bar{w}_s = \bar{w}_{s-1}(1 + \bar{x}(\bar{w}_{s-1})) - \bar{a}_s \quad s = 1, \dots, n \quad (\otimes\otimes)$$

if and only if (Par).

*Proof:* Assume (Par). Then

$$\begin{aligned}
\bar{w}_{s-1}(1 + \bar{x}(\bar{w}_{s-1})) - \bar{a}_s &= [\text{for (Par)}] = C^{s-1} - C_{s-1} + x(w_{s-1})w_{s-1} - a_s - \text{SVA}_s \\
&= [\text{for (12)}] = C^{s-1}(1 + i(C^{s-1})) - C_{s-1}(1 + i(C_{s-1})) - a_s \\
&= C^s - C_s \\
&= \bar{w}_s.
\end{aligned}$$

Assume now  $\otimes\otimes$ . We have then

$$\begin{aligned}
\bar{w}_{s-1} + \bar{x}(\bar{w}_{s-1})\bar{w}_{s-1} - \bar{a}_s &= \bar{w}_s \\
&= C^s - C_s \\
&= C^{s-1}(1 + i(C^{s-1})) - C_{s-1}(1 + i(C_{s-1})) - a_s \\
&= [\text{for (12)}] = C^{s-1} - C_{s-1} + x(w_{s-1})w_{s-1} - a_s - \text{SVA}_s \\
&= C^{s-1} - C_{s-1} + x(w_{s-1})w_{s-1} - \bar{a}_s \\
&= \bar{w}_{s-1} + x(w_{s-1})w_{s-1} - \bar{a}_s
\end{aligned}$$

whence

$$\bar{x}(\bar{w}_{s-1})\bar{w}_{s-1} = x(w_{s-1})w_{s-1}$$

which implies (Par).

(Q.E.D.)

The result of Lemma 6.2 enables us to give  $\bar{w}_s$  the interesting interpretation of outstanding capital for the shadow project  $\bar{P}$  at the two-valued rate  $\bar{x}(\bar{w}_{s-1})$ . To complete the parallelism between  $P$  and  $\bar{P}$  we give the following definitions:

**Definition 5:** If (Twin), the Economic Value Added of  $P$  is the product

$$\text{EVA}_s := w_{s-1}(x(w_{s-1}) - i(C_{s-1})) \quad (15a)$$

which means one of the following:

$$\text{EVA}_{P,N} := w_{s-1}(x_P - i_N)$$

$$\text{EVA}_{N,P} := w_{s-1}(x_N - i_P)$$

$$\text{EVA}_{P,P} := w_{s-1}(x_P - i_P)$$

$$\text{EVA}_{N,N} := w_{s-1}(x_N - i_N).$$

**Definition 6:** If (Twin) and (Par), the Economic Value Added of  $\bar{P}$  (or *shadow* EVA) is the product

$$\overline{\text{EVA}}_s := \bar{w}_{s-1}(\bar{x}(w_{s-1}) - i(C_{s-1})) \quad (15b)$$

which means one of the following:

$$\overline{\text{EVA}}_{P,N} := \bar{w}_{s-1}(\bar{x}_P - i_N)$$

$$\overline{\text{EVA}}_{N,P} := \bar{w}_{s-1}(\bar{x}_N - i_P)$$

$$\overline{\text{EVA}}_{P,P} := \bar{w}_{s-1}(\bar{x}_P - i_P)$$

$$\overline{\text{EVA}}_{N,N} := \bar{w}_{s-1}(\bar{x}_N - i_N).$$

These Definitions are based on the following way of reasoning: At the beginning of each period, one can invest the capital  $w_{s-1}$  ( $\bar{w}_{s-1}$  for  $\bar{P}$ ) either at the rate  $x(w_{s-1})$  ( $\bar{x}(\bar{w}_{s-1})$  for  $\bar{P}$ ) or at the rate  $i(C_{s-1})$  (the same for  $\bar{P}$ ). Accepting the first alternative her profit will be  $x(w_{s-1})w_{s-1}$  ( $\bar{x}(\bar{w}_{s-1})\bar{w}_{s-1}$  for  $\bar{P}$ ); the other course of action will leave her with  $i(C_{s-1})w_{s-1}$  ( $i(C_{s-1})\bar{w}_{s-1}$  for  $\bar{P}$ ). The residual income is then given by the difference between the two, whence we obtain (15).

*Remark 6.1:* The definition of EVA in (15a) is unambiguous only if (Twin), otherwise we could wonder whether we have to use  $i(C_{s-1})$  or  $i(C^{s-1})$ . The same is true for the definition of  $\overline{\text{EVA}}$  in (15b), in which case we must add the assumption (Par), otherwise  $\bar{w}_{s-1}$  is not economically interpretable as outstanding capital for  $\bar{P}$ .

This brings about the following:

**Theorem 6.1** *If (Twin) and (Par), then the periodic Systemic Value Added coincides with the Economic Value Added of the shadow project, that is*

$$\text{SVA}_s = \overline{\text{EVA}}_s \quad \text{for every } s. \quad (16a)$$

In this case we have

$$\begin{aligned} \text{SVA} = \text{NFV} &= \sum_{s:\bar{w}_{s-1}>0, C_{s-1}<0}^n \bar{w}_{s-1}(\bar{x}_P - i_N) + \sum_{s:\bar{w}_{s-1}<0, C_{s-1}>0}^n \bar{w}_{s-1}(\bar{x}_N - i_N) \\ &+ \sum_{s:\bar{w}_{s-1}>0, C_{s-1}>0}^n \bar{w}_{s-1}(\bar{x}_P - i_P) + \sum_{s:\bar{w}_{s-1}<0, C_{s-1}<0}^n \bar{w}_{s-1}(\bar{x}_N - i_N) \end{aligned} \quad (16b)$$

*Proof:* Using Lemma 6.1 and Lemma 6.2 we have

$$\bar{w}_{s-1}(1 + \bar{x}(\bar{w}_{s-1})) - \bar{a}_s = \bar{w}_{s-1}(1 + i(C_{s-1})) - a_s$$

which implies

$$\bar{x}(\bar{w}_{s-1})\bar{w}_{s-1} - \text{SVA}_s = i(C_{s-1})\bar{w}_{s-1}$$

whence

$$\text{SVA}_s = \bar{w}_{s-1}(\bar{x}(\bar{w}_{s-1}) - i(C_{s-1})) = [\text{for (Definition 6)}] = \overline{\text{EVA}}_s \quad (\text{Q.E.D.})$$

*Remark 6.2:* According to Theorem 6.1 the Systemic Value Added model we have obtained by means of a systemic argument resembles Stewart's decomposition: We just have to use the concept of Economic Value Added and decompose the shadow of  $P$ . Thus, the SVA model can be interpreted as a derivation of the EVA model. Likewise, the EVA model itself can be seen as a derivation of the SVA model:  $P$  is the shadow project of some other project  $P'$  and then the periodic  $\text{EVA}_s$  of  $P$  coincides with the periodic Systemic Value Added of  $P'$ .

**Lemma 6.3.** *If (Twin), then*

$$\text{SVA}_1 = \text{EVA}_1 \quad (18a)$$

and

$$\text{SVA}_s = \text{EVA}_s + i(C_{s-1}) \sum_{k=1}^{s-1} \text{EVA}_k (1 + i(C))^k, (s-1) \quad \text{for every } s > 1. \quad (18b)$$

*Proof:* We have

$$\begin{aligned} \text{SVA}_1 &= [\text{for (12)}] = x(w_0)w_0 + i(C_0)C_0 - i(C^0)C^0 \\ &= [\text{for (Twin)}] = x(w_0)w_0 - i(C_0)\bar{w}_0 \\ &= [\text{for } w_0 = \bar{w}_0] = w_0(x(w_0) - i(C_0)) \\ &= [\text{for (15a)}] = \text{EVA}_1. \end{aligned}$$

If  $s > 1$ , we get

$$\begin{aligned} (C^{s-1} - C_{s-1}) &= [\text{for (Twin)}] = w_0(1 + i(C))^{0, (s-1)} - \sum_{k=1}^{s-1} a_k(1 + i(C))^{k, (s-1)} \\ &= w_0(1 + i(C))^{0, (s-1)} - \sum_{k=1}^{s-1} (w_{k-1}(1 + x(w_{k-1})) - w_k)(1 + i(C))^{k, (s-1)} \\ &= [\text{rearranging terms}] = w_{s-1} - \sum_{k=1}^{s-1} w_{s-1}(x(w_{k-1}) - i(C_{k-1})). \end{aligned} \quad (19)$$

We have then

$$\begin{aligned}
\text{SVA}_s &= [\text{for (Twin)}] = x(w_{s-1})w_{s-1} - i(C_{s-1})(C^{s-1} - C_{s-1}) \\
&= [\text{for (19)}] = w_{s-1}(x(w_{s-1}) - i(C_{s-1})) + i(C_{s-1}) \sum_{k=1}^{s-1} w_{k-1}(x(w_{k-1}) - i(C_{k-1}))(1 + i(C))^{k,(s-1)} \\
&= [\text{for (15a)}] = \text{EVA}_s + i(C_{s-1}) \sum_{k=1}^{s-1} \text{EVA}_k(1 + i(C))^{k,(s-1)}.
\end{aligned}$$

(Q.E.D.)

**Theorem 6.2** *If (Twin) , then*

$$\sum_{k=1}^s \text{SVA}_s = \sum_{k=1}^s \text{EVA}_s(1 + i(C))^{k,s} \quad \text{for every } s \geq 1. \quad (20)$$

*Proof:* Using induction, we have, for  $s=1$ ,  $\text{SVA}_1 = \text{EVA}_1$  (Lemma 6.3). Suppose (20) holds for  $s=m$ . Then,

$$\begin{aligned}
\sum_{k=1}^{m+1} \text{SVA}_k &= \sum_{k=1}^m \text{SVA}_k + \text{SVA}_{m+1} \\
&= [\text{for Lemma 6.3}] = \sum_{k=1}^m \text{SVA}_k + \text{EVA}_{m+1} + i(C_m) \sum_{k=1}^m \text{EVA}_k(1 + i(C))^{k,m} \\
&= [\text{by ind. hyp.}] = \sum_{k=1}^m \text{EVA}_k(1 + i(C))^{k,m} + \text{EVA}_{m+1} + i(C_m) \sum_{k=1}^m \text{EVA}_k(1 + i(C))^{k,m} \\
&= \sum_{k=1}^{m+1} \text{EVA}_k(1 + i(C))^{k,m+1} \quad (Q.E.D.)
\end{aligned}$$

*Remark 6.3:* Let  $P_p$  be a project and let  $P_{p+1}$  be its shadow project (then  $P_{p-1}$  denotes a project such that  $P_p$  is its shadow project). Denote with  $\text{SVA}_s^p$  and  $\text{EVA}_s^p$  the periodic Systemic Value Added and the Economic Value Added of  $P_p$  respectively. From Theorems 6.1 and 6.2 and Lemma 6.3 we have

$$\begin{aligned}
\text{SVA}_s^p &= \text{EVA}_s^{p+1} \\
\text{SVA}_s^{p-1} &= \text{EVA}_s^p \\
\text{SVA}_s^p &= \text{SVA}_s^{p-1} + i(C_{s-1}) \sum_{k=1}^{s-1} \text{SVA}_k^{p-1}(1 + i(C))^{k,(s-1)} \\
\text{EVA}_s^{p+1} &= \text{EVA}_s^p + i(C_{s-1}) \sum_{k=1}^{s-1} \text{EVA}_k^p(1 + i(C))^{k,(s-1)} \\
\text{SVA} &= \sum_{s=1}^n \text{SVA}_s^p = \sum_{s=1}^n \text{EVA}_s^{p+1} = \sum_{s=1}^n \text{EVA}_s^p(1 + i(C))^{s,n}.
\end{aligned}$$

## 7. The EVA Theorems

In this section we provide some results on the decomposition of NFVs which include, among others, all results obtained by P&S (though stated in our systemic parlance) but we have a different outlook and our proofs simply stem from the just introduced concept of Systemic Value Added.

**Proposition 7.1** *If for all  $s$   $C_s$  and  $C^s$  are both nonnegative or both nonpositive, then (Twin).*

*Proof:* From Definition 1 (and pointing out that  $i(0)$  can be defined *ad libitum*) (Q.E.D.).

**Proposition 7.2.** *If  $E_0=0$ , then (Twin) and  $C_s=-\bar{w}_s$  for all  $s$ .*

*Proof:* We have  $C^s=0$  for all  $s$  and  $-C_s=C^s - C_s=\bar{w}_s$  for all  $s$ . Further, we have that  $C^s=0$  for all  $s$  implies that for all  $s$   $C_s$  and  $C^s$  are both nonnegative or both nonpositive, whence  $(i_P, i_N)$  is a twin-pair (Proposition 7.1). (Q.E.D.)

**Proposition 7.3.** *If  $E_0=0$ , then  $\text{NFV}=E_n=C_n$*

*Proof:* If  $E_0=0$ , we have  $E^n=0$ , so that

$$\begin{aligned}\text{NFV} &= E_n - E^n \\ &= E_n \\ &= C_n + w_n = C_n\end{aligned}$$

(Q.E.D.)

**Theorem 7.1** *Assume (Twin). Then Peccati's model can be generalized in a two-rate capitalization of periodic shares  $G_s$  so that*

$$G_s = \text{EVA}_s (1 + i(C))^{s,n}. \quad (21a)$$

*In this case, we have*

$$\sum_{s=1}^n G_s = \sum_{s=1}^n \text{EVA}_s (1 + i(C))^{s,n} = \text{NFV}$$

*or, more explicitly,*

$$\begin{aligned}\text{NFV} &= \sum_{s:w_{s-1}>0, C_{s-1}<0}^n w_{s-1}(x_P - i_N)(1 + i(C))^{s,n} + \sum_{s:w_{s-1}<0, C_{s-1}>0}^n w_{s-1}(x_N - i_P)(1 + i(C))^{s,n} \\ &+ \sum_{s:w_{s-1}>0, C_{s-1}>0}^n w_{s-1}(x_P - i_P)(1 + i(C))^{s,n} + \sum_{s:w_{s-1}<0, C_{s-1}<0}^n w_{s-1}(x_N - i_N)(1 + i(C))^{s,n}\end{aligned} \quad (21b)$$

*Proof:* Applying Peccati's argument we have

$$\begin{aligned}G_s &= -w_{s-1}(1 + i(C))^{(s-1),n} + (w_s + a_s)(1 + i(C))^{s,n} \\ &= w_{s-1}(x(w_{s-1}) - i(C_{s-1}))(1 + i(C))^{s,n} \\ &= [\text{for (15a)}] = \text{EVA}_s(1 + i(C))^{s,n}\end{aligned}$$

and

$$\begin{aligned}
\sum_{s=1}^n G_s &= \sum_{s=1}^n \text{EVA}_s (1 + i(C))^{s,n} \\
&= [\text{for Theorem 6.2}] = \sum_{s=1}^n \text{SVA}_s \\
&= \text{SVA} \\
&= E_n - E^n \\
&= \text{NFV}
\end{aligned}$$

(Q.E.D.)

**Corollary 7.1** *If  $C_0 = -a_0$  then (21) holds.*

*Proof:* From Proposition 7.2 and Theorem 7.1.

(Q.E.D.)

**Lemma 7.1.** *If  $E_0 = 0$ , then, for all  $s$ ,*

$$\bar{x}(\bar{w}_{s-1}) = \bar{x}_P \quad \text{iff} \quad i(C_{s-1}) = i_N. \quad (22)$$

*Proof:* If  $E_0 = 0$  then  $\bar{w}_s = -C_s$  for all  $s$  (Proposition 7.2). Then, for all  $s$ ,

$$\bar{x}(\bar{w}_{s-1}) = \bar{x}_P$$

if and only if

$$0 \leq \bar{w}_{s-1} = -C_{s-1}$$

if and only if

$$i(C_{s-1}) = i_N$$

(Q.E.D.)

**Theorem 7.2.** *Assume  $E_0 = 0$ . Peccati's model can be generalized in a two-rate capitalization of periodic shares so that*

$$G_s = \text{EVA}_{P,N}^\pi \text{EVA}_{N,P}^{1-\pi} (1 + i(C))^{s,n}, \quad (23a)$$

(with  $\pi$  being a boolean variable), if and only if (Par).

In this case, we have

$$\begin{aligned}
\text{NFV} &= \sum_{s=1}^n G_s \\
&= \sum_{s:w_{s-1} > 0} w_{s-1} (x_P - i_N) (1 + i(C))^{s,n} + \sum_{s:w_{s-1} < 0} w_{s-1} (x_N - i_P) (1 + i(C))^{s,n} \quad (23b)
\end{aligned}$$

*Proof:*  $E_0 = 0$  implies (22) (Lemma 7.1) and (Twin) (Proposition 7.2). (Twin) implies (21) (Theorem 7.1).



Suppose first (Par). (22) and (Par) imply

$$x(w_{s-1}) = x_P \quad \text{iff} \quad i(C_{s-1}) = i_N.$$

The latter and (21) imply (23).

Suppose now that (23) holds. Then

$$x(w_{s-1}) = x_P \quad \text{iff} \quad i(C_{s-1}) = i_N.$$

The latter and (22) imply (Par).

(Q.E.D.)

*Remark 7.1:* It is worthwhile noting that Theorem 7.2 implies P&S Theorem. The latter is proved by the authors by means of a rule on the factorization of particular bivariate polynomials. As we see, there is no need of such a rule. Our proof rests on the economic concept of Systemic Value Added and does not depend on formal properties of polynomials, deriving from the more general result in Theorem 7.1.

**Definition 7:**  $P$  is said to be a Soper project if for all  $s$   $x(w_{s-1})=x_P$ .  $\bar{P}$  is said to be a Soper project if for all  $s$   $\bar{x}(\bar{w}_{s-1})=\bar{x}_P$

**Theorem 7.3.** *If  $C_0=-a_0$ , ( $i_N$ -Twin) and (SP), then*

$$\text{NFV} = \sum_{s=1}^n w_{s-1}(x_P - i_N)(1 + i_N)^{n-s}. \quad (24)$$

*Proof:* ( $i_N$ -Twin) implies (Twin). (Twin) implies

$$\sum_{s=1}^n \text{SVA}_s = \sum_{s=1}^n \text{EVA}_s(1 + i(C))^{s,n}$$

(Theorem 6.2). (SP) and ( $i_N$ -Twin) imply  $\text{EVA}_s=w_{s-1}(x_P - i_N)$  and  $(1 + i(C))^{s,n}=(1 + i_N)^{n-s}$ . Then, (24) holds, since

$$\text{NFV} = E_n - E^n = \sum_{s=1}^n \text{SVA}_s. \quad (\text{Q.E.D.})$$

The above Theorem mirrors Proposition 6 of P&S (p.179). Note that our proof does not make use of the first assumption, so we can relax it and state the following more general:

**Theorem 7.4.** *If ( $i_N$ -Twin) and (SP), then*

$$\text{NFV} = \sum_{s=1}^n w_{s-1}(x_P - i_N)(1 + i_N)^{n-s}.$$

**Proposition 7.4.** *If  $C_0=-a_0$  and (Par), then*

$$E_n = C_n = \sum_{s:w_{s-1}>0}^n w_{s-1}(x_P - i_N)(1 + i(C))^{s,n} + \sum_{s:w_{s-1}<0}^n w_{s-1}(x_N - i_P)(1 + i(C))^{s,n} \quad (25)$$

*Proof:* Use Proposition 7.3 and Theorem 7.2.

(Q.E.D.)

**Proposition 7.5.** *If  $C_0 = -a_0$  and (Par), then*

$$E_s = \sum_{k:w_{k-1}>0} w_{k-1}(x_P - i_N)(1 + i(C))^{k,s} + \sum_{k:w_{k-1}<0} w_{k-1}(x_N - i_P)(1 + i(C))^{k,s} \quad (26)$$

where  $1 \leq k \leq s$ .

*Proof:*  $C_0 = -a_0$  is equivalent to  $E_0 = 0$ , which implies  $E^s = 0$  for all  $s$ . We have then

$$\text{SVA}_s = (E_s - E_{s-1})$$

so that

$$E_s = E_0 + \sum_{k=1}^s \text{SVA}_k = \sum_{k=1}^s \text{SVA}_k. \quad (27)$$

$E_0 = 0$  implies (Twin) (Proposition 7.2), which in turn implies (20) (Theorem 6.2).  $E_0 = 0$  implies (22) (Lemma 7.1). (22) and (Par) imply

$$x(w_{s-1}) = x_P \quad \text{iff} \quad i(C_{s-1}) = i_N.$$

The latter, (20) and (27) imply (26).

(Q.E.D.)

**Proposition 7.6.** *If  $C_0 = -a_0$ , then*

$$E_s = E_{s-1}(1 + i(C_{s-1})) + w_{s-1}(x(w_{s-1}) - i(C_{s-1})) = E_{s-1}(1 + i(C_{s-1})) + \text{EVA}_s. \quad (28)$$

*Proof:* Since  $E^s = C^s = 0$  for all  $s$ , we have

$$\begin{aligned} E_s &= E_{s-1} + \text{SVA}_s = E_{s-1} + x(w_{s-1})w_{s-1} + i(C_{s-1})C_{s-1} - i(C^{s-1})C^{s-1} \\ &= E_{s-1} + x(w_{s-1})w_{s-1} + i(C_{s-1})C_{s-1} \\ &= E_{s-1} + i(C_{s-1})(C_{s-1} + w_{s-1}) - i(C_{s-1})w_{s-1} + x(w_{s-1})w_{s-1} \\ &= E_{s-1}(1 + i(C_{s-1})) + w_{s-1}(x(w_{s-1}) - i(C_{s-1})) \\ &= E_{s-1}(1 + i(C_{s-1})) + \text{EVA}_s \end{aligned} \quad (29)$$

(Q.E.D.)

**Corollary 7.2.** *If  $C_0 = -a_0$ , then  $\text{SVA}_s = E_s - E_{s-1} = \text{EVA}_s + i(C_{s-1})E_{s-1}$ .*

*Proof:* Straightforward from the proofs of Propositions 7.5 and 7.6.

(Q.E.D.)

*Remark 7.2:* Corollary 7.2 informs us that when  $E_0 = 0$  the periodic Systemic Value Added is the net profit for period  $s$  and the difference between  $\text{SVA}_s$  and  $\text{EVA}_s$  is given by the interest gained on the initial net worth  $E_{s-1}$ . Proposition 7.6 provides us with the diachronic evolution of the investor's wealth: Note that in terms of  $\text{SVA}_s$  we have an "accounting-flavored" equation according to which the end-of-period net worth is given by the sum of the initial net worth and the net profit (which in this case coincides with  $\text{SVA}_s$ ). In terms of  $\text{EVA}_s$  we have an equation according to which the sum  $E_{s-1}$  must be compounded at the rate  $i(C_{s-1})$  and the  $\text{EVA}_s$  must be added to it in order to obtain the end-of-period wealth. The latter relation is typical of financial calculus: We can see  $E_s$  as the value of an account  $E$  providing us with the periodic value of the whole wealth. Actually, such an investment

is *The Investment* pre-eminently, where the investor invests  $E_{s-1}$  at the rate  $i(C_{s-1})$  and at the end of period the  $EVA_s$  is paid into account  $E$  (see (28)). Here different perspectives are at work: One is based on accounting-like reasoning, measuring the profit and summing to it the initial capital invested (initial wealth+profit), the other one is founded on financial calculus (or, to say better, it is NFV-based), measuring the differential gain and summing to it the compounded initial wealth (compounded wealth+residual income). This implies that we can have decompositions of cash flows streams based on the one or the other perspective. The SVA perspective does not rest on capitalization process and on profitability indexes, it just relies on computation of initial capital invested and net profit. This provides an integration between accounting and capital budgeting. But in the systemic perspective we do not use accounting as such, we use the way accounting represents economic facts, that is by means of a systemic approach. As we shall see, this is even more satisfying from a diachronic point of view (see section 9.).

**Corollary 7.3.** *If  $C_0 = -a_0$  and (Par), then*

$$E_s = E_{s-1}(1 + i(C_{s-1})) + EVA_{P,N}^\pi EVA_{N,P}^{1-\pi} \quad (30)$$

where  $\pi$  is a boolean variable.

*Proof:*  $C_0 = -a_0$  implies (28) (Proposition 7.6) and (22) (Lemma 7.1). (22), (Par) and (28) imply (30). (Q.E.D.)

**Proposition 7.7.** *If  $C_0 = -a_0$  and (Par), then  $w_s \geq 0$  implies  $C_s \leq 0$ . Likewise,  $w_s \leq 0$  implies  $C_s \geq 0$ .*

*Proof:* The first hypothesis implies (22) (Lemma 7.1). (22), (Par) and  $w_s \geq 0$  imply  $i(C_{s-1}) = i_N$ , that is  $C_s \leq 0$ . The second part is analogous. (Q.E.D.)

**Corollary 7.4.** *If  $C_0 = -a_0$ , (Par) and  $E_s > 0$  for some  $s$ , then*

$$-C_s < w_s < 0 < E_s$$

or

$$C_s \leq 0 < E_s \leq w_s.$$

*Proof:*  $E_s > 0$  implies  $w_s > -C_s$ . Then, if  $w_s < 0$  we have

$$-C_s < w_s < 0;$$

if  $w_s \geq 0$  we have  $C_s \leq 0$  (Proposition 7.7) so that

$$C_s \leq 0 < E_s = w_s + C_s \leq w_s.$$

(Q.E.D.)

## 8. The shadow Theorems.

We now show some results which are companions of the previous ones in that the shadow  $EVA_s$  is essential and plays the same role  $EVA_s$  has played in the former section. Capitalization is now superfluous, since we are adopting an accounting way of reasoning (remember Remark 7.2).

**Proposition 8.1.** *If  $C_0 = -a_0$  and (Par), then (16) holds.*

*Proof:* From Proposition 7.2 and Theorem 6.1.

(Q.E.D.)

The following is the counterpart of Theorem 7.2:

**Theorem 8.1.** *Assume  $E_0=0$ . Then*

$$\text{SVA}_s = \overline{\text{EVA}}_{P,N}^\pi \overline{\text{EVA}}_{N,P}^{1-\pi} \quad (31a)$$

(with  $\pi$  being a boolean variable), if and only if (Par).

In this case, we have

$$\begin{aligned} \text{NFV} &= \text{SVA} \\ &= \sum_{s:\overline{w}_{s-1}>0}^n \overline{w}_{s-1}(\overline{x}_P - i_N) + \sum_{s:\overline{w}_{s-1}<0}^n \overline{w}_{s-1}(\overline{x}_N - i_P) \end{aligned} \quad (31b)$$

*Proof:*  $E_0=0$  implies (22) (Lemma 7.1). Suppose first (Par).  $E_0=0$  and (Par) imply (16) (Proposition 8.1). (16) and (22) imply (31). Suppose now that (31) holds. Then (16) holds *a fortiori*. Hence,

$$\begin{aligned} \overline{w}_{s-1}(\overline{x}(\overline{w}_{s-1}) - i(C_{s-1})) &= \overline{\text{EVA}}_s \\ &= \text{SVA}_s \\ &= [\text{for (12)}] = x(w_{s-1})w_{s-1} + i(C_{s-1})C_{s-1} - i(C_{s-1})C_{s-1} \\ &= [\text{for (Twin)}] = x(w_{s-1})w_{s-1} - i(C_{s-1})\overline{w}_{s-1} \end{aligned}$$

whence

$$\overline{x}(\overline{w}_{s-1})\overline{w}_{s-1} = x(w_{s-1})w_{s-1}$$

which implies (Par).

(Q.E.D.)

**Lemma 8.1.** *If both (SP) and ( $\overline{SP}$ ), then (Par). In particular,  $x(w_{s-1})=x_P$  and  $\overline{x}(\overline{w}_{s-1})=\overline{x}_P$ .*

*Proof:* From Definitions 4 and 7.

(Q.E.D.)

Now we state the counterpart of Theorem 7.3. The latter requires  $P$  to be a Soper project. But in the systemic approach we are provided with two projects, project  $P$  and its shadow  $\overline{P}$ . What about  $\overline{P}$  in order to reach a decomposition analogous to (24)? For  $\overline{P}$  to be worth of being named “shadow” of  $P$ , we expect it to adhere to project  $P$ ’s features. In fact, we have the following:

**Theorem 8.2.** *If  $C_0=-a_0$ , ( $i_N$ -Twin), (SP) and ( $\overline{SP}$ ), then*

$$\text{NFV} = \sum_{s=1}^n \overline{w}_{s-1}(\overline{x}_P - i_N). \quad (32)$$

*Proof:* (SP) and ( $\overline{SP}$ ) imply (Par) (Lemma 8.1).  $C_0=-a_0$  and (Par) imply (31) (Theorem 8.1). (31) and ( $i_N$ -Twin) imply (32).

(Q.E.D.)

We can relax the first assumption as the proof can be reshaped as follows:

*Proof.* ( $i_N$ -Twin) implies (Twin). (SP) and ( $S\bar{P}$ ) imply (Par), with  $\bar{x}(\bar{w}_{s-1})=\bar{x}_P$  (Lemma 8.1). (Par) and (Twin) imply (16) (Theorem 6.1).  $\bar{x}(\bar{w}_{s-1})=\bar{x}_P$ , (16) and ( $i_N$ -Twin) imply (32).

We have then proved:

**Theorem 8.3.** *If ( $i_N$ -Twin), (SP) and ( $S\bar{P}$ ), then*

$$\text{NFV} = \sum_{s=1}^n \bar{w}_{s-1}(\bar{x}_P - i_N)$$

which is the counterpart of Theorem 7.4. The companion of Proposition 7.4 is the following:

**Proposition 8.2.** *If  $C_0=-a_0$  and (Par), then*

$$E_n = C_n = \sum_{s:\bar{w}_{s-1}>0}^n \bar{w}_{s-1}(\bar{x}_P - i_N) + \sum_{s:\bar{w}_{s-1}<0}^n \bar{w}_{s-1}(\bar{x}_N - i_P) \quad (33)$$

*Proof:* Use Proposition 7.3 and Theorem 8.1.

(Q.E.D.)

The counterpart of Proposition 7.5 is:

**Proposition 8.3.** *If  $C_0=-a_0$  and (Par), then*

$$E_s = \sum_{k:\bar{w}_{k-1}>0} \bar{w}_{k-1}(\bar{x}_P - i_N) + \sum_{k:\bar{w}_{k-1}<0} \bar{w}_{k-1}(\bar{x}_N - i_P) \quad (34)$$

where  $1 \leq k \leq s$ .

*Proof:*  $C_0=-a_0$  implies  $E^s=0$  for all  $s$ . We have then

$$\text{SVA}_s = (E_s - E_{s-1})$$

so that

$$E_s = E_0 + \sum_{k=1}^s \text{SVA}_k = \sum_{k=1}^s \text{SVA}_k.$$

$C_0=-a_0$  and (Par) imply

$$\text{SVA}_k = \bar{w}_{k-1}(\bar{x}_P - i_N)^\pi \bar{w}_{k-1}(\bar{x}_N - i_P)^{1-\pi}$$

(Theorem 8.1). We have then

$$E_s = \sum_{k=1}^s \text{SVA}_k = \sum_{k:\bar{w}_{k-1}>0} \bar{w}_{k-1}(\bar{x}_P - i_N) + \sum_{k:\bar{w}_{k-1}<0} \bar{w}_{k-1}(\bar{x}_N - i_P)$$

with  $1 \leq k \leq s$ .

(Q.E.D.)

The counterpart of Corollary 7.3 is:

**Proposition 8.4.** *If  $C_0=-a_0$  and (Par), then*

$$E_s = E_{s-1} + \overline{\text{EVA}}_{P,N}^\pi \overline{\text{EVA}}_{N,P}^{1-\pi} \quad (35)$$

where  $\pi$  is a boolean variable.

*Proof:* We have  $E^s=0$  for all  $s$  so that

$$E_s = E_{s-1} + \text{SVA}_s.$$

$C_0=-a_0$  and (Par) imply (31a) (Theorem 8.1), so that

$$E_s = E_{s-1} + \text{SVA}_s = E_{s-1} + \overline{\text{EVA}}_{P,N}^\pi \overline{\text{EVA}}_{N,P}^{1-\pi}$$

(Q.E.D.)

As you see, in the SVA model you just have to sum the initial period net worth to project  $\overline{P}$ 's  $\text{EVA}_s$ , whereas in the NFV-based models you have to compound the net worth and then sum it to project  $P$ 's EVA.

As for Corollary 7.4, in the SVA model it becomes:

**Proposition 8.5.** *If  $C_0=-a_0$ , (Par) and  $E_s>0$  for some  $s$ , then*

$$\overline{w}_s < w_s < 0 < E_s$$

or

$$-\overline{w}_s \leq 0 < E_s \leq w_s.$$

*Proof:* As we know,  $C_0=-a_0$  implies  $C_s=-\overline{w}_s$ . The conclusion follows from Corollary 7.4

(Q.E.D.)

It is worthwhile noting that in case of zero net worth, account  $C$  acts as the shadow project, as the following Proposition shows:

**Proposition 8.6.** *If  $E_0=0$ , then  $C=\overline{P}$  and  $E_s=w_s - \overline{w}_s$  so that*

<u>Uses</u>		<u>Sources</u>	
$w_s$		$\overline{w}_s$	
		$E_s$	(36)

*Proof:* Obvious, since  $C_s=-\overline{w}_s$ .

(Q.E.D.)

## Remarks

The SVA model introduced in this paper allows for a decomposition of a project's Net Final Value differing from Stewart's as well as Peccati's and P&S's. The latter three are NFV-based models, whereas the concept of Systemic Value Added is, so to say, "accounting-flavored". Regardless of the assumption on the rates EVA and SVA rely on two different interpretations of the notion of residual income. To clarify this issue, assume, for sake of convenience,  $i(C_{s-1}) = i(C^{s-1}) = i$ ,  $x(w_{s-1}) = x$  and  $\overline{x}(w_{s-1}) = \overline{x} = x \frac{w_{s-1}}{\overline{w}_{s-1}}$ . Also,  $x \neq i$  (otherwise the decision process is an idle issue). As regards EVA, Stewart's implicit way of reasoning is the following: At the beginning of period  $s$  the capital invested in the project is  $w_{s-1}$ ; the evaluator could invest the sum at an alternative rate equal to  $i$ . This implies that

she is gaining  $xw_{s-1}$  while renouncing to the sum  $iw_{s-1}$ . The difference between the two alternatives is the differential gain of one alternative over the other, which represents the residual income for period  $s$ :

$$\text{EVA}_s = w_{s-1}(x - i).$$

As for SVA, the line of argument stems from the fact that cash flows can be seen as sources (if negative) or uses (if positive) of funds. It is possible to describe the decision maker's financial system diachronically by drawing up a sequence of double-entry sheets for each alternative. In the case the undertaking of  $P$  is selected (9a) holds, otherwise (9b) is used. If the evaluator chooses investment  $P$  then her profit for period  $s$  will be  $E_s - E_{s-1}$ , if she instead invests money at the alternative rate  $i$ , her profit will be  $E^s - E^{s-1}$ . The difference between the two is the residual income of period  $s$ . The formal consequences of the two lines of argument have been analyzed in the previous sections. In particular, the Systemic Value Added enables us to partition the NFV with no need of capitalization. Actually,  $\text{EVA}_s$  is money referred to time  $s$  so that compounding (discounting) is required to compute the NFV (NPV) of the project. On one side, the SVA model is incompatible with the EVA model, since  $\text{SVA}_s \neq \text{EVA}_s$  (only in overall terms they coincide giving rise to the NFV). The two models provide different information and in our opinion the selection of which one must be used depends just on the information the evaluator wishes to have. On the other side, the SVA model and the EVA model can be seen as two sides of the same medal through introduction of the concept of shadow project: Each project has a shadow project and is itself a shadow project of some other project. This is the reason why we can see the SVA as an EVA or the EVA as a SVA. In this sense the SVA model suggests an interpretation of residual income such that project  $P$ 's periodic residual income is obtained by means of computation of an Economic Value Added, not referred to project  $P$  itself, but to its shadow.

The concept of shadow project is essential. The outstanding balance  $\bar{w}_s$  is the sum the evaluator could invest at the rate  $i$  at the beginning of period  $s$ . Undertaking the project, i.e. investing  $w_{s-1}$  at the rate  $x$  the decision maker renounces to the sum  $i\bar{w}_{s-1}$  in order to receive the sum  $xw_{s-1}$ , which can be written as  $\bar{x}\bar{w}_{s-1}$ . The difference is the residual income. Thus, economically, the shadow project represents the alternative course of action. Alternative (i) and (ii) can be rewritten as follows:

- (I) undertaking project  $P$
- (II) undertaking project  $\bar{P}$ .

Consequently, we can represent the corresponding financial systems as

(I)		(II)					
<u>Uses</u>		<u>Sources</u>		<u>Uses</u>		<u>Sources</u>	
$C_{s-1}$		$E_{s-1}$		$C_{s-1}$		$E^{s-1}$	
$w_{s-1}$				$\bar{w}_{s-1}$			
(I)				(II)			
<u>Uses</u>		<u>Sources</u>		<u>Uses</u>		<u>Sources</u>	
$C_s$		$E_s = E_{s-1} + iC_{s-1} + (xw_{s-1})$		$C_s$		$E^s = E^{s-1} + iC_{s-1} + (i\bar{w}_{s-1})$	
$w_s$				$\bar{w}_s$			

(37)

for time  $s-1$  and  $s$  respectively.

The decision maker must select the preferred alternative;  $C_s$  is shared by both courses of action, but (I) ensures a profit equal to  $xw_s + iC_{s-1}$  whereas (II) offers a profit of  $i\bar{w}_s + iC_{s-1}$  (note that sheet (II) in (37) is just (9b) in a different form). Stewart and Peccati, as well as P&S, implicitly replace  $\bar{w}_{s-1}$  by  $w_{s-1}$  in (II) so that  $E_s = E^s$  and  $SVA_s$  boils down to  $EVA_s$  (that is  $xw_{s-1} - i\bar{w}_{s-1}$  turns to  $xw_{s-1} - iw_{s-1}$ ). From this point of view, this replacement brings about some problems. Actually, if we substitute  $\bar{w}_{s-1}$  for  $w_{s-1}$  for all  $s$ , we have, for  $s^*$  fixed,

$$E^{s^*-1} = C_{s^*-1} + w_{s^*-1} \quad (38a)$$

$$E^{s^*} = C_{s^*} + w_{s^*}; \quad (38b)$$

but (38a) implies

$$E^{s^*} = C_{s^*-1}(1+i) + w_{s^*-1}(1+i) \quad (38c)$$

since (II) implies that the net worth is invested at the rate  $i$ . (38b) and (38c) are incompatible since

$$w_{s^*-1}(1+x) \neq w_{s^*-1}(1+i).$$

This whimsical result is followed by the ambiguous idea of compounding  $EVA_s$  to obtain the NFV. As we have seen, the latter can be seen as the sum of un compounded periodic Systemic Values Added or, alternatively, as the sum of compounded periodic Economic Values Added. In a sense, the SVA enables us to overlook capitalization. This is a striking result, as this is contrary to basic financial calculus. Yet, it is perfectly consistent with an accounting outlook. Further, if we sum the periodic net profits we obtain the difference  $E_n - E_0$ , which is, financially speaking, the total interest gained on the net worth invested at time 0. As a matter of fact this would suggest that an accounting-flavored approach with no compounding can be helpful in appraising investments, provided that we use cash values rather than accounting values. Note also that the NFV can be seen as the sum of un compounded shadow EVAs. We could then call the SVA model as a “shadow EVA model”. With the plain EVA model we have sums that refer to time  $s$  so they must be compounded with the factor  $(1+i)^{n-s}$ . This seems to distort the process of imputation:  $(1+i)^{n-s}$  collects interest that is generated in periods subsequent to period  $s$ . Should we regard them as belonging to the residual income of period  $s$ ? This seems to be the idea of Peccati, according to whom the  $s$ -th quota of the NFV is  $G_s$ , which refers to time  $n$ . So then, is  $EVA_s$  or  $EVA_s(1+i)^{n-s}$  to be ascribed to period  $s$ ? In the latter case, we impute interest that, as we have said, is generated in other periods. In the former case, we have  $n$  periodic residual incomes whose sum do not lead to the overall residual income (NFV): We would have that the sum of the parts does not coincide with the whole. The SVA model does not have such drawbacks. It accomplishes a perfect partition, for the sum of periodic residual income generate, as one expects, the overall residual income.

We do not state here that the EVA model is incorrect and that the SVA is correct. The inconsistency we have shown is such only because we are in a systemic-diachronic outlook, so the evolution of the financial system is relevant. Further, it is in our opinion a mere convention to adopt one or the other. The index the decision maker has to use depends on the information she wishes to obtain, that is on the notion of residual income she is inclined to adopt.

In our decomposition model, as well as in P&S’s model, there are some conventional elements that are worth pointing out. As we know from TRM, *op.cit.*, there are infinite internal pair  $(x_P, x_N)$  so that  $w_n=0$ : which one is the pair to be selected for decomposing the Net Final Value in order to achieve a *correct* residual income? P&S do not say anything about it. In our opinion the choice is conventional,



only in some simple cases being straightforward (if the project is a Soper project then we have a unique internal rate of return  $x_P$ ). If  $C_0 = -a_0$  we could rely on the fact that

$$\text{NFV}(x_P, x_N) = w_n = -a_0(1 + x(w))^{0,n} + \sum_{s=1}^n a_s(1 + x(w))^{s,n} = 0.$$

We know that NFV implicitly defines  $x_P$  as function of  $x_N$  and viceversa. So we can pick alternatively  $x_P := i$  or  $x_N := i$  so that

$$x_N = x_N(x_P) = x_N(i)$$

or

$$x_P = x_P(x_N) = x_P(i).$$

We have then

$$\text{NFV}(x_P, x_N) = \text{NFV}(i, x_N(i)) = 0 \quad (39a)$$

or

$$\text{NFV}(x_P, x_N) = \text{NFV}(x_P(i), i) = 0. \quad (39b)$$

The decision maker must choose (39a) or (39b) so that  $\text{EVA}_s$ ,  $\text{SVA}_s$  and  $\overline{\text{EVA}}_s$  will be univocally determined. The choice of one of the two is not immediate and future researches could be devoted to the problem of selecting the most significant one from an economic point of view. Also, if we assume  $i_P \neq i_N$ , as we have done in this paper, there arise other problems: unless ( $i_N$ -twin) or ( $i_P$ -twin), there exist some periods in which  $i(C_s) = i_P$  and some other periods in which  $i(C_s) = i_N$ . Then the evaluator does not know which is the one to be chosen in (39). Also, the idea of assuming a unique market rate  $i$  is *economically* different from our assumption of a pair  $(i_P, i_N)$ . In the latter case we are assuming that funds can be borrowed at a rate  $i_N$  differing from the reinvestment rate  $i_P$ . To be precise, we are assuming that account  $C$  is a sort of current account where different rates apply depending on the sign of  $C$ , whereas TRM rest on the assumption of a unique opportunity cost of capital (obviously, if  $i_P = i_N$  we get back to TRM's model). It is also worthwhile noting that if  $(i_P, i_N)$  is not a twin-pair, the analysis of TRM cannot be applied, since

$$\text{NFV}(x_P, x_N) \neq w_n$$

so that the concept of Net Final Value does not coincide with the concept of project balance at time  $n$ . There arises the problem of defining what an internal pair is: is it a pair such that  $\text{NFV} = 0$  or is it a pair such that  $w_n = 0$ ?

Operationally, if we adopt Stewart's point of view many such problems can be overlooked. According to an EVA approach, investors forecast the value of the capital invested  $w_s$  and the periodic rate of return for period  $s$ ,  $x_s$ . No problem of existence or uniqueness of rates of return arises. So doing, we simply have

$$\text{EVA}_s = w_{s-1}(x_s - i)$$

or, with debt,

$$\text{EVA}_s = w_{s-1}(x_s - i) + D_{s-1}(i - \delta_s)$$

where  $\delta_s$  is the ROD referred to period  $s$  ( $i$  is sometimes taken as variable over time, so that  $i$  is replaced by  $i_s$ ). As for  $\text{SVA}_s$  we have

$$\text{SVA}_s = x_s w_{s-1} - i_s(C^s - C_s)$$

or, with debt,

$$SVA_s = x_s w_{s-1} - \delta_s D_{s-1} - i_s (C^s - C_s).$$

Theoretically, it can be interesting to investigate the behavior of NFV in relation to  $w_n$  when  $E_0 \neq 0$  and try to provide some rules in order to select the most significant pair  $(x_P, x_N)$ , so that the meaning of  $EVA_s$  and  $SVA_s$  is economically transparent. However, the selection is natural if  $(x_P, x_N)$  is fixed *a priori*, which occurs whenever the project is connected to an account  $w$  (e.g. for a financial agreement) where cash flows are invested in or withdrawn from: The value of such an account is obviously  $w_s$ . In such a case, when  $a_s$  is positive,  $w$  reduces by the sum  $a_s$  while  $C$  increases by the same sum; when  $a_s$  is negative,  $w$  raises by the sum  $a_s$  and  $C$  decreases by the same sum. The decomposition is then straightforward as the four rates to be used are fixed *a priori* and univocally determined for each period by the sign of the two accounts.

### Conclusions.

This paper has several goals: first of all, it aims at showing that Stewart's model, Peccati's model, Pressacco and Stucchi's model bear strong relations one another from a formal point of view; secondly, it generalizes the concept of EVA by including it in a TRM framework where two-valued rates are used. Thirdly, some results on the decomposition of the NFV of a project are shown, including all results obtained by P&S. Fourthly, the concept of shadow project enables us to compute NFVs and partition them through a systemic (i.e. accounting-flavored) outlook, so that we obtain an index (SVA) which does not rest on capitalization and therefore seems to formally trespass the basic rules of financial calculus. Each result has its own shadow counterpart so that decomposition can be illustrated by focusing on the shadow project. Further, the idea of a shadow project gives us the opportunity to see the SVA model as an EVA model, where we compute the shadow project's EVA to decompose a project's NFV. Actually, the SVA model seems to be more satisfying from the point of view of the financial system's evolution and from the point of view of a correct decomposition. As for the latter, the EVA model provides us with quotas whose sum do not offer the whole, as we would expect; as for the former, the EVA model shows some inconsistencies, which we have not dwelt on. The SVA model solves these problems by introducing the SVA (and the shadow EVA) and offsetting capitalization, while from an evolutionary perspective the financial system is correctly grasped in double-entry sheets, which record the activation of the accounts at each time.

Finally, a striking result is, in our opinion, that we have provided a framework for integration between capital budgeting and accounting (not accounting itself, but the philosophy of accounting). This integration is done through gradual steps which leads us to change from the financial-calculus formula

$$NFV = -a_0 + \sum_{s=1}^n a_s (1 + i(C))^{s,n}$$

based on cash flows to the financial-calculus formula

$$NFV = \sum_{s=1}^n EVA_s (1 + i(C))^{s,n}$$

which is based on periodic residual income. Hence, we offset capitalization and offer the systemic formula

$$NFV = \sum_{s=1}^n SVA_s$$

which is based on differential net profits. The latter can be in turn rewritten in terms of Economic Value Added by means of the shadow project, so that

$$\text{NFV} = \sum_{s=1}^n \overline{\text{EVA}}_s$$

which avoids capitalization.<sup>7</sup>

Both theoretical and operational developments can be investigated in future researches. Theoretically, more relations among SVA, EVA,  $\overline{\text{EVA}}$  can be investigated, as well as connections between the Net Final Value of  $P$  and the Net Final Value of  $\overline{P}$ , and the concept of internal pair should be clarified, as we said in the latter section. Further, the conceptual difference between the EVA model and the SVA model should, in our opinion, attract attention: The notion of residual income is ambiguous, at least two interpretations can be proposed. Are other interpretations possible? Are they mere conventions? And if they are, can we say they are not *arbitrary* conventions? These and other questions deserve to be answered.

Operationally, rules should be given to forecast the correct SVA and thus to draw up a correct sequence of double-entry sheet. Future researches could be addressed to extending the results by allowing for many  $C$ -type accounts and/or a portfolio of projects and/or multiple loan contracts, so that (9a) is replaced by

Uses	Sources
$K_s^1$	$D_s^1$
$K_s^2$	$D_s^2$
...	...
$K_s^p$	...
$w_s^1$	...
$w_s^2$	...
...	$D_s^m$
$w_s^q$	$E_s$

and (9b) is replaced by

Uses	Sources
$K_s^1$	$E^s$
$K_s^2$	
...	
$K_s^p$	

where  $K_s^j$  is the value of account  $K^j$ ,  $w_s^r$  is the outstanding capital of project  $P^r$  and  $D_s^l$  is the outstanding debt of loan contract  $D^l$ ,  $j = 1, \dots, p$ ,  $r = 1, \dots, q$ ,  $l = 1, \dots, m$ .

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<sup>7</sup>All these formulas hold under the assumptions we have studied in the previous sections. But the idea of replacing  $\text{EVA}_s$  with  $\text{SVA}_s$  as a periodic residual income is independent from any such assumptions: It only rests on a different cognitive perspective which adopts a different interpretation of the notion of residual income.

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