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Long-run Welfare under Externalities in Consumption, Leisure, and Production: A Case for Happy Degrowth vs. Unhappy Growth

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Abstract

In this paper we contribute to the debate on the relationship between growth and well-being by examining an endogenous growth model where we allow for externalities in consumption, leisure, and production. We analyze three regimes: a decentralized economy where each household makes isolated choices without considering their external effects, a planned economy where a myopic planner fails to recognize both leisure and consumption externalities but recognizes production externalities, and a planned economy with a fully informed planner. We first compare the balanced growth paths under the three regimes and then we numerically investigate the transition to the optimal balanced growth path. We provide a number of findings. First, in a decentralized economy growth or labor (or both) are greater than in the regime with a fully informed planner, and hence are sub-optimal from a welfare standpoint. Second, a myopic intervention which overlooks consumption and leisure externalities leads to more growth and labor than in both the decentralized and the fully informed regime. Third, we provide a case for happy degrowth: a transition to the optimal balanced growth path that is associated with downscaling of production, a reduction in private consumption, and an ongoing increase in leisure and well-being.

JEL classification: Q13; E62; H21; H23.
Keywords: degrowth; endogenous growth; consumption externalities; leisure externalities; production externalities.

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1 Introduction

The importance of economic growth for enhancing well-being in high-income countries has been challenged from different perspectives (see, e.g., Victor, 2008). Two of these, in particular, have received increasing attention in recent years: the struggle for relative social position and the decline of relational activities. The first has been widely studied in the literature on social status and consumption externalities (see, e.g., Clark et al., 2008, and references therein) while the second has been investigated especially in the literature on subjective well-being and relational goods (see, e.g., Bruni and Porta, 2007, and references therein).

We contribute to the debate on the relationship between growth and well-being by developing and analyzing an endogenous growth model in which we include externalities in consumption, leisure, and production. Externalities in consumption are modeled as stemming from the dynamics of relative social position while externalities in leisure are modeled as stemming from relational activities. Externalities in production are assumed to arise from aggregate increasing returns. In particular, the inclusion of positive production externalities – beyond being the cornerstone of endogenous growth models – allows us to be conservative with regard to the relationship between growth and well-being.

The aim of the paper is twofold. On the one hand, we try to shed light on the complex interrelation between those three kinds of externalities in order to evaluate the desirability of growth versus degrowth and labor versus leisure. On the other, we move a first step towards the investigation of transitional degrowth paths which are compatible with welfare enhancement in an endogenous growth framework. In doing so, we have been inspired by some recent contributions on the so-called “negative endogenous growth”, which suggest that economic growth might be fueled by the disruption of social or environmental assets (see e.g., Bartolini and Bonatti, 2002, 2008). With respect to this literature we add further sources of externalities and, more importantly, we ask a different question: is there any room for happy degrowth?

We consider three economic regimes: a decentralized economy where every household takes isolated choices without considering their external effects, a planned economy where a myopic planner fails to recognize both leisure and consumption externalities but recognizes production externalities, and a planned economy with a fully informed planner. In the first part of our analysis, we identify and compare the balanced growth paths under the three regimes. We find that in a decentralized economy either growth or labor (or both) must be greater than in the regime with a fully informed planner, implying that they are sub-optimal from a welfare standpoint. We also show that a myopic intervention which overlooks consumption and leisure externalities leads to more growth and labor than in both the decentralized regime and the fully informed one. This means that the situation in such a case is likely to be even worse than in the decentralized regime. If we consider that in the current policy debate production externalities are certainly recognized while consumption and leisure externalities are seldom even referred to, our finding suggests that there exists a real risk of biases toward growth and labor. In the second part of our analysis, we numerically explore the transition to the optimal balanced path starting from sub-optimal ones. More precisely, we provide a case for happy degrowth: under a set of reasonable parameter values, the transition to the optimal balanced growth path is associated with production downscale, reduction in private consumption, and...

\footnote{See also Antoci et al. (2011) for the impact of technical progress.}
and ongoing increase in leisure and well-being.

The paper is organized as follows. The next section briefly introduces the related literature and our theoretical framework. Section 3 illustrates the model and provides the analysis of the balanced growth path under the three regimes. Section 4 presents the numerical analysis and discusses the main results of the model. Section 5 provides our final remarks.

2 Theoretical framework and related literature

2.1 Consumption externalities through relative social standing

The first explicit and formal recognition of consumption externalities in economics can be traced back to the work of Veblen (1899) and Pigou (1903). However, the modern approach to consumption externalities has been mostly influenced by Duesenberry (1949) who translated Veblen’s main insights into standard economic language. Duesenberry’s basic idea is that individuals care not only about their own level of consumption but also about their consumption relative to that of people in their reference group. This way of modeling consumption externalities is often referred to as the interdependent preferences approach, and has attracted a number of established scholars (see, e.g., Pollack, 1976, Hayakawa and Venieris, 1977, Corneo and Jeanne, 1997, Clark and Oswald, 1996, 1998, Hollander, 2001, Hayakawa, 2000). In particular, the core of the interdependent preferences approach consists of assuming that (or appropriately postulating preference relations so that) an individual’s utility function depends positively on the individual’s level of consumption and negatively on the consumption of people in the individual’s reference group. The introduction of this negative externality is meant to represent the effects on well-being due to the comparison with other individuals.

Another representative instance of the modeling of consumption externalities – although more focused on the idea of social status – is found in the seminal paper by Frank (1985) on the demand for positional and nonpositional goods. The distinctiveness of this approach is the fact that individuals are assumed to get utility both from consumption of standard nonpositional goods and from their rank in the distribution of consumption of positional goods (a similar modeling can be found in, e.g., Corneo and Jeanne, 1998, Robson, 1992, Dierer, 2001, Hopkins and Kornienko, 2004, 2009). Since there is a fixed number of social positions, a negative consumption externality arises: spending more on positional goods allows a better position to be attained but this comes at the cost of lowering someone else’s position.

With reference to such alternative modeling approaches, Bilancini and Boncinelli (2008a) have shown that it can make a substantial difference for the model predictions whether one posits that what matters is just rank or, instead, one assumes that other cardinal features matter such as the distance from average consumption or the ratio between one’s own consumption and the average. In order to keep the model analytically tractable, in this paper we model consumption externalities

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2 Frank (1985) explicitly acknowledges that the distinction between positional and nonpositional goods was introduced by Hirsch (1976).

3 Bilancini and Boncinelli (2008b) show that different modeling choices of social status can lead to opposite predictions on the relationship between income inequality and waste in conspicuous consumption. Bilancini and Boncinelli (2009) show that a similar issue arises with regard to the relationship between wage inequality and waste in conspicuous consumption.
in a way that is similar to the one applied in Duesenberry (1949). More precisely, we assume that utility depends on the ratio between an individual’s own consumption and the average consumption of other individuals. However, we stress that the issue raised by Bilancini and Boncinelli (2008a) is not relevant here since what is crucial to our results is that rising average consumption has a negative effect on individuals’ utility, no matter how or why. We also note that under different formulations our model would be less analytically tractable, requiring a greater reliance on numerical simulations.

Of particular interest for our paper is the stream of literature that focuses on the macroeconomic consequences of the “keeping up with the Joneses” assumption – i.e., individuals struggling to keep up with the social position of their representative neighbor. Ljungqvist and Uhlig (2000) analyze the impact of consumption externalities on the effect of short-run macroeconomic stabilization policy. Dupor and Liu (2003) define different forms of consumption externalities and explore their relationship with equilibrium over-consumption. The paper by Cooper et al. (2001) deserves special attention in this regard as the first attempt to introduce such externalities in a growth model. More precisely, Cooper et al. (2001) allow for both positive production externalities – arising from an R&D sector – and negative consumption externalities – arising from the struggle for a higher social position. Interestingly, they find that positive GDP growth can go with negative utility growth. The paper by Liu and Turnovsky (2005) also studies the optimal growth in the presence of both consumption and production externalities, showing that the type of distortion depends on both the type and relative strength of externalities.

Our paper follows the spirit of this stream of literature, in particular of Cooper et al. (2001) and Liu and Turnovsky (2005). We are somewhat less general on the specification of both preferences and technology, but we add an important further source of externality – i.e., relational activities – that interacts with production and consumption externalities. Moreover, we do not assume that individuals try to strictly “keep up with the Joneses”, but only that an individual’s marginal utility of own consumption increases as others’ consumption increases.4 We remark that this choice, which is made again to preserve analytical tractability, works against the existence of a case for happy degrowth, and hence makes our results stronger. Indeed, the existence of a path of happy degrowth is more easily obtained when individuals try to strictly “keep up with the Joneses” because in such a case an increase in average material consumption has the further (indirect) negative effect on well-being of inducing individuals to consume more.

Finally, we recall that consumption externalities are not just a hypothesis of economic theorists. Actually, there are several empirical studies documenting the existence of consumption externalities and suggesting that they play an important role in determining people’s well-being (see, e.g., Senik, 2005, for a survey). For instance, Solnick and Hemenway (1998) conducted a survey study where approximately fifty percent of the respondents stated that they would be happy to give up half of their real purchasing power in order to have a higher relative standing. McBride (2001) exploits US survey data to study the same issue and finds similar evidence. Using experimental survey methods Alpizar et al. (2005) obtain qualitatively similar results. Interestingly, they also show that consumption externalities are present in activities traditionally thought to be nonpositional like vacations and insurances. Furthermore, in an important study about the determinants of

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4 As well explained in Liu and Turnovsky (2005), when labor is elastic the assumption of “keeping up with the Joneses” requires that the ratio between the marginal utility of consumption and the marginal utility of leisure increases when others’ consumption increases.
reported happiness in Britain and US, Blanchflower and Oswald (2004) find that both absolute and relative position matter (see Ferrer-i-Carbonell, 2005, for a similar analysis on German panel data). Clark and Senik (2010) provide evidence from large-scale survey data on both the intensity and direction of income comparisons for Europeans, showing that comparisons are considered to be at least somewhat important by three-quarters of the population and that they are associated with lower levels of subjective well-being.

2.2 Leisure externalities through relational activities

The relevance of relational activities as determinants of individual well-being has been stressed by various scholars (see, e.g., Gui and Sugden, 2005, Uhlaner, 2009, and references therein), but the basic idea can be traced back to the work of Gui (1987) and Uhlaner (1989). Their point is that the fruits of many human activities are not enjoyed through the market system but, instead, through social interactions. In particular, relational activities are performed by the agents involved in social interactions, possibly using pre-existing social ties and producing new or modifying old social ties. Relational activities which positively affect well-being include companionship, empathic communication, emotion sharing, psychological support, solidarity, love relations, etc. On the small scale, these activities are jointly produced by those involved in family relationships and networks of friendship. On the large scale, they are jointly produced by those participating in social gatherings such as public meetings, festivals, sports, and club meetings.

A relational activity produces relational goods which, in turn, are consumed by those who take part in the relational activity. Relational goods can be seen as a specific kind of local public good (e.g., Becchetti et al., 2008). The “public” character of relational goods is due to the non-rival feature of their consumption: all participants in the relational activity which has produced the relational good can consume it.5

There is a growing body of empirical evidence supporting the idea that more and better relational activities go with a greater well-being. A series of work by John Helliwell and his co-authors (see, e.g., Helliwell, 2006, Helliwell and Huang, 2007, Helliwell et al., 2009) has shown that social capital in general is positively correlated with subjective well-being – measured as reported happiness or reported life satisfaction. Further evidence more specific to relational activities is provided by Helliwell and Putnam (2004) for the US, by Bruni and Stanca (2008) for many of the countries surveyed in the World Values Survey, by Powdthavee (2008) for the UK, Bartolini et al. (2011) for the US between 1975 and 2004, and Bartolini et al. (2010) for Germany between 1996 and 2007. Evidence of a causal relationship going from relational activities to subjective well-being is provided by Becchetti et al. (2008).

Importantly, both individual participation in relational activities and existing social ties generate positive externalities, raising the utility from the consumption of relational goods and contributing to create further consumption opportunities for others. Moreover, relational activities feed back into

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5 In this paper we focus on relational activities and do not consider the wider concept of social capital – which may encompass further objects such as general trust and confidence in institutions. Moreover, we abstract from potential positive spillovers of social capital on economic growth since this would make the case for happy degrowth easier, against our conservative approach (for an analysis considering such spillovers see, e.g., Roseta-Palma et al., 2010). For a more detailed discussion of the relationship between social capital and economic growth see Neira et al. (2009) and references therein. In particular, see Beugelsdijk and Smulders (2009) for a recent analysis including endogenous labor and distinguishing between bridging and bonding social capital.
existing social ties, strengthening them.\textsuperscript{6}

In this paper we try to capture the role of relational activities by appropriately modeling individuals’ utility functions and by specifying the dynamics of existing social ties. More precisely, we assume that individuals obtain utility from the consumption of relational goods which, in turn, are jointly produced by the existing stock of social ties and by the amount of individual leisure, which is considered as a proxy of the time spent in relational activities. Furthermore, we assume that the accumulation of social ties depends on the average leisure time, which again proxies the amount of relational activities which have a positive feedback on existing social ties.

2.3 Production externalities through accumulation of knowledge

Although the notions of increasing returns and endogenous technical progress were both present in the Classical theory of growth, it was not until Marshall (1890) that the distinction between external and internal economies was clearly set out. Such a distinction allows the co-existence of increasing returns at an aggregate level and decreasing returns at a micro level, which is crucial for a competitive equilibrium to exist.\textsuperscript{7} Marshall’s contribution, together with that by Young (1928), gave what is now a well recognized representation of the engine of economic growth: aggregate positive production externalities allow the limits to economic growth due to diminishing returns at the firm level to be overcome.

The presence of positive production externalities was later formalized by Arrow (1962), who developed an optimal growth theory based on learning-by-doing.\textsuperscript{8} More precisely, spillovers accruing from learning-by-doing are formalized by including, among the arguments of the production function at the firm level, the aggregate capital stock. The idea, well explained by Romer (1986), is that the aggregate stock of capital proxies the stock of available knowledge at a certain time, which affects the productivity of private investments. In particular, the rate of investment may not decrease as the stock of capital grows because the accumulation of knowledge acts as an external effect enhancing the rate of return on capital.

The more recent literature on endogenous growth has explored a variety of interpretations and modeling approaches to the knowledge accumulation process: capital accumulation triggers an extension of the technological capabilities of the economy (Romer, 1986), creation of new knowledge by one firm enlarges the set of production possibilities of other firms (Romer, 1990), investment in human capital improves the productivity of labor (Lucas, 1988), investment in R&D induces technological innovation which, in turn, allows the production of higher value goods (Aghion and Howitt, 1992, Grossman and Helpman, 1991). In this paper, we follow Romer (1986), who considers

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\textsuperscript{6}To fix ideas, consider an individual, say Eve, who watches something on TV that she likes. Of course, Eve gets some utility from such an isolated activity. In the absence of other individuals, Eve has no relational goods to consume and her utility comes entirely from watching TV. Suppose now that a friend of Eve’s joins her, say Adam, and suppose that Adam spends a couple of hours watching TV with Eve, during which the two interact a lot and have a good time. Eve consumes the relational good produced by the relational activity and by the existing friendship, increasing her utility beyond that obtained by watching TV alone. Adam consumes the same relational good and he also gets utility from watching TV. Moreover, the friendship between Eve and Adam becomes stronger, making the next joint session watching TV both more likely and more enjoyable.

\textsuperscript{7}For a discussion on the relation between classical and endogenous growth theory see for instance D’Alessandro and Salvadori (2008) and references therein.

\textsuperscript{8}See also Sheshinski (1967).
the average stock of capital as a proxy of the level of knowledge of the economy. Alternatively, the stock of capital can be interpreted as a proxy for intra-industry spillovers due to knowledge diffusion (Barro and Sala-i-Martin, 1992).

Notwithstanding the theoretical interest in production externalities, the available empirical evidence has not yet established what relevance production externalities have to real economies. Using industry data on value added as a measure of output, Caballero and Lyons (1990, 1992) establish that there is a strong positive correlation between sectoral productivity and aggregate activity, interpreting this finding as evidence of productive externalities. However, Basu and Fernald (1995) argue that, under imperfect competition, value-added data predictably lead one to find large apparent externalities even if such externalities do not exist. Once one corrects the estimates for such an issue, evidence is mixed: gross output regressions suggest that there are no externalities, while modified doubly-deflated value-added regressions suggest that there are large external effects. Burnside (1996) provides at least three additional arguments against the evidence on the existence of externalities, concluding that available evidence suggests that the typical U.S. manufacturing industry displays constant returns with no external effects. Benaroch (1997) follows the approach of Basu and Fernald (1995) augmenting the model with a geographical component and obtaining evidence of production externalities in Canada. In particular, his study shows that at the two-digit SIC level there are increasing returns to scale in Canadian manufacturing industries that are national in nature and external to both the industry and the province.

Although there is mixed evidence of the presence of production externalities, in our model we allow for positive production externalities. The reason for this choice is that we want to be conservative about conditions that work against the possibility of happy degrowth.

3 The model

3.1 Assumptions

Households. The population of this economy is composed by a fixed number \( N \) of infinitely lived and identical households.\(^9\) Each household is endowed with a unit of time that has to be allocated to either leisure or labor. Labor is used, together with accumulated capital goods, to produce a material good which can be immediately consumed or accumulated. The instantaneous utility function of household \( i \in \{1, \ldots, N\} \) is:

\[
U_i(c_i, \bar{c}, l_i, x_i) = \frac{(c_i \bar{c}^{-\gamma} l_i^{\omega} x_i^{\theta})^{1-\sigma}}{1-\sigma},
\]

where \( l_i \) is \( i \)'s leisure, \( c_i \) is \( i \)'s consumption of material goods, \( \bar{c} \equiv \sum_{j=1}^{N} c_j/N \) is average consumption of material goods, and \( x_i \) is \( i \)'s consumption of relational goods. We set \( \omega, \theta > 0 \) and \( 0 < \gamma < 1 \), meaning that both \( i \)'s leisure and \( i \)'s consumption of relational goods positively affect utility while average material consumption negatively affects utility. Note that the net effect of material consumption is positive. We also set \( \sigma > 1 \) to ensure concavity and hence solvability of the optimization problem.

\(^9\)In order to have meaningful consumption and leisure externalities there must exist more than one individual. However, the assumption that agents are identical and infinitely lived is only motivated by analytical tractability. Having heterogeneous or finitely lived agents that care for their children would not affect the quality of our results.
Technology. The production of material goods employs as inputs both capital and labor. We introduce positive production externalities by assuming a production technology in line with Romer (1986). Formally, household $i$’s production function for material goods is:

$$y_i = A(\bar{k})k_i^\alpha (1 - l_i)^{(1-\alpha)},$$

where $k_i$ is $i$’s capital employed in production and $(1 - l_i)$ is $i$’s labor employed in production. Note that we impose constant return to scale. We also set $0 < \alpha < 1$, meaning that the marginal productivity of each private factor of production is positive and decreasing. The factor $A(\bar{k})$ is meant to account for the positive externality in production. More precisely, following Romer (1986) we set:

$$A(\bar{k}) = A\bar{k}^{1-\alpha},$$

where $\bar{k} = \sum_{j=1}^{N} k_j/N$. This configuration of external effects in production ensures constant returns to scale in the accumulable factor (i.e., capital) at the aggregate level. Finally, we denote with $r$ the remuneration rate of capital and with $w$ the wage. Hence, in a competitive market equilibrium we would have $r = \alpha A(\bar{k})k_i^\alpha (1 - l_i)^{(1-\alpha)}$ and $w = (1 - \alpha)A(\bar{k})k_i^\alpha (1 - l_i)^{-\alpha}$.

Relational goods and social ties. The production of relational goods employs as inputs both leisure and the stock of social ties. More precisely, household $i$’s production function for relational goods is:

$$x_i = l_i^{1-\xi} V^\xi,$$

where $V$ is the current stock of social ties in the economy. We set $0 < \xi < 1$, meaning that marginal productivity of both leisure and social ties is positive and decreasing. Note that there we impose constant returns to scale. Note also that we model the stock of social ties as an aggregate public good which can be accessed by every household.

Furthermore, the stock of social ties is the result of past time spent by anybody who engaged in relational activities, minus the natural decay of social ties. To model this we posit that the accumulation of social ties depends on the average level of leisure in the economy, denoted by $\bar{l} = \sum_{i=1}^{N} l_i/N$, minus a certain amount of decayed ties. Formally, we model the dynamics of the stock of social ties as follows:

$$\dot{V} = B\bar{l} - \zeta V,$$

where $B > 0$ measures the contribution of average leisure and $\zeta > 0$ measures the depreciation rate of the stock of social ties. This simple formalization takes into account both the coordination issue which is intrinsic to the production of social ties and the fact that people must systematically take care of social ties in order not to have them fade away.

Finally, in the light of equations (1) and (4), we can rewrite the utility function as:

$$u(c_i, \bar{c}, l_i, V) = \frac{(c_i \bar{c}^{-\gamma} l_i^\phi V^\eta)^{1-\sigma}}{1 - \sigma},$$

where, in order to simplify notation, we set $\phi \equiv \omega + (1 - \xi)\phi$ and $\eta \equiv \theta \xi$. Equation (6) makes the dependence of utility on social ties apparent, hence the existence of positive externalities in leisure.
3.2 A decentralized economy

In a decentralized equilibrium every household maximizes the stream of utility from time zero to infinity under its budget constraint. In doing so each household behaves atomistically, taking as given the aggregate stock of capital, average material consumption, average leisure and the stock of social ties. Assuming a preferences discount rate $\rho > 0$, the household’s maximization problem is:

$$\max \int_0^\infty \left( c_i \bar{c}\phi V^\phi V^{1-\sigma} \right) ^\frac{1}{1-\sigma} e^{-\rho t} dt$$ (7)
subject to the budget constraint:

$$\dot{k}_i = rk_i + w(1 - l_i) - \delta k_i - c_i,$$ (8)

and an initial stock of capital. By applying Pontryagin’s Maximum Principle, in the symmetric equilibrium – i.e., where all households behave identically – the following conditions must hold:

$$g_{k,d} = \dot{k}_d = A(1 - l_d)^{(1 - \alpha)} - \frac{c_d}{k_d},$$ (9)

$$g_{c,d} = \frac{\dot{c}_d}{\dot{k}_d} = \alpha A(1 - l_d)^{(1 - \alpha)} - \frac{(\rho + \delta)}{1 - (1 + \gamma)(1 - \sigma)},$$ (10)

$$c_d = \frac{(1 - \alpha)A(1 - l_d)^{-\alpha}}{\phi} l_d,$$ (11)

where the subscript $d$ indicates that the variable refers to the decentralized economy. Because of symmetry, $k_{i,d} = k_d$ and $c_{i,d} = c_d$ for all $i$. From (9), (10), and (11) we can obtain a level of leisure which ensures $g_{c,d} = g_{k,d}$ and therefore results in a balanced growth path. In principle, a balanced growth path might not exist or there might be more than one balanced path. However, as we show in the Appendix A, a unique balanced growth path exists with $l_d \in (0, 1)$ whenever $g_{c,d} = g_{k,d} > 0$. In the Appendix A we also show that the same holds for the other two regimes.

3.3 A myopically planned economy

We now consider the maximization problem of a myopic social planner that internalizes only production externalities, but fails to recognize both leisure and consumption externalities. As in the case of the decentralized economy, the stock of social ties is taken as given as well as the average level of material consumption. However, external effects in production are taken into account – i.e., equation (3) – so that the accumulation of capital is given by:

$$\dot{k} = Ak(1 - l)^{(1 - \alpha)} - \delta k - c.$$ (12)

The myopic social planner maximizes (7) under (12), starting from a positive stock of physical capital. In this case, the symmetric equilibrium must satisfy the following conditions:
\[
g_{k,m} \equiv \frac{\dot{k}_m}{k_m} = A(1 - l_m)^{(1-\alpha)} - \delta - \frac{c_m}{k_m}, \tag{13}
\]
\[
g_{c,m} \equiv \frac{\dot{c}_m}{c_m} = \frac{A(1 - l_m)^{(1-\alpha)} - (\rho + \delta)}{1 - (1 - \gamma)(1 - \sigma)}, \tag{14}
\]
\[
\frac{c_m}{k_m} = \frac{(1 - \alpha)A(1 - l_m)^{-\alpha} l_m}{\phi}, \tag{15}
\]
where the subscript \(m\) indicates that the variable refers to the case of an economy managed by a myopic planner. Because of symmetry, \(k_{i,m} = k_m\) and \(c_{i,m} = c_m\) for all \(i\).\(^{10}\)

We can compare the balanced growth paths of a decentralized and a myopically planned economy. Since the myopic planner recognizes production externalities, both investment in private capital and supply of labor are larger than in the decentralized economy, leading to a greater balanced growth rate. This is evident by noting that equations (9) and (11) are the same as equation (13) and (15), while equation (14) induces a \(g_c\) that is greater than that induced by equation (10) for every level of leisure which is smaller than 1. Intuitively, a myopic social planner that takes production externalities into account imposes more work, more consumption of material goods, less leisure and, hence, less consumption of relational goods. This means that an economy managed by a myopic planner which overlooks consumption and leisure externalities ends up growing more than a decentralized economy.

### 3.4 Non-myopically planned economy

Finally, we consider the maximization problem of a fully informed social planner, i.e., a planner that recognizes all externalities in this economy and which is aware of the dynamics of the stock of social ties. Such a fully informed social planner does not maximize (7), but instead maximizes the following:

\[
\max \int_0^\infty \frac{(c_i^{1-\gamma} l_i^{\phi} V^n)^{1-\sigma}}{1 - \sigma} e^{-\rho t} dt \tag{16}
\]
subject to both (12) and (5). Therefore, in a symmetric equilibrium the following equations must hold (see Appendix B for details):

\(^{10}\)As for the decentralized economy, in the Appendix A, we show that if for \(0 < l_m < 1\) there exists a balanced growth path with \(g_{c,m} = g_{k,m} > 0\), then the balanced growth path is unique.
\[ g_{k,p} \equiv \frac{\dot{k}_p}{k_p} = A (1 - l_p)^{(1-\alpha)} - \delta - \frac{c_p}{k_p}, \quad (17) \]
\[ g_{c,p} \equiv \frac{\dot{c}_p}{c_p} = \frac{A (1 - l_p)^{(1-\alpha)} - (\rho + \delta)}{1 - (1 - \gamma)(1 - \sigma)}, \quad (18) \]
\[ c_p = \frac{(1 - \gamma)(1 - \alpha)A (1 - l_p)^{-\alpha} l_p \{ (\rho + \zeta) + \Psi [A (1 - l_p)^{1-\alpha} - (\delta + \rho)] \}}{\eta \zeta + \phi \{ (\rho + \zeta) + \Psi [A (1 - l_p)^{1-\alpha} - (\delta + \rho)] \}}, \quad (19) \]
\[ \Psi = - \frac{(1 - \gamma)(1 - \sigma)}{1 - (1 - \gamma)(1 - \sigma)^4}, \quad (20) \]

where the subscript \( p \) indicates that the variable refers to the case of an economy managed by a fully informed planner. Again, \( k_{i,p} = k_p \) and \( c_{i,p} = c_p \) for all \( i \), because of symmetry.\(^{11}\)

We can compare the balanced growth paths of the economy when it is managed by a myopic planner and by a fully informed planner. Since the fully informed planner recognizes all externalities and also takes into account the dynamics of the stock of social ties, both investment in private capital and supply of labor are smaller than those under the myopic social planner, leading to a smaller balanced growth rate. This is evident from the fact that equation (18) is the same as equation (14), while equations (13) and (15) induce a \( g_k \) that is greater than that induced by equations (17) and (19) for every level of leisure which is smaller that 1. Intuitively, compared with the myopic social planner a fully informed social planner correctly evaluates that material consumption is less useful and leisure more useful. This means that an economy managed by a myopic social planner which overlooks consumption and leisure externalities ends up growing more than an economy managed by a fully informed social planner but with households experiencing a lower well-being. Therefore, the myopic planner is likely to induce more growth than both a decentralized economy and a fully informed social planner, but at the cost of lowering household welfare.

The comparison between the balanced growth path of an economy managed by a fully informed social planner and a decentralized economy is less straightforward. A decentralized economy may have higher balanced growth, with more material consumption, more work and less relational activities. However, balanced growth under a fully informed social planner may also be higher, with more material consumption and less relational goods. Growth may also be lower in the decentralized economy, but with leisure and relational goods also lower. The actual relative characteristics of the two balanced growth paths depend on the relative intensity of the three externalities as well as on the dynamics of the stock of social ties. Intuitively, if production externalities are much stronger than consumption and leisure externalities, then balanced growth will be greater under a fully informed planner. If, instead, consumption and leisure externalities are stronger than production externalities, then balanced growth will be greater in the decentralized economy. Furthermore, the equilibrium level of leisure and relational goods is more likely to be higher under the fully informed planner if leisure contributes more to the accumulation of social ties. What cannot happen is that under the fully informed planner both balanced growth and the level of leisure are smaller than in

\(^{11}\)In the Appendix A, we show that also in the non-myopically planned economy if for \( 0 < l_p < 1 \) there exists a balanced growth path with \( g_{c,p} = g_{k,p} > 0 \), then the balanced growth path is unique.
the decentralized economy. Finally, of course under the fully informed planner household welfare is greater than in a decentralized economy.

4 Long-run balanced growth and transition

4.1 Long-run steady state

Figure 1 shows, for the three regimes considered, a numerical simulation of equilibrium conditions on the growth rate of both capital stock and material consumption as functions of leisure time. The dashed curve represents the growth rate of material consumption in the decentralized economy, as describe by equation (10). The solid curve represents the growth rate of material consumption in the myopically and non-myopically planned economy, as described by equations (14) and (18), respectively. The dotted curve represents the capital growth rate in the decentralized and myopically planned economy, as described by equations (9) and (13) in the light of equations (11) and (15), respectively. The dash-dotted curve represents the capital growth rate in the non-myopically planned economy, as described by equation (17) in the light of (19). The points highlighted with a circle, i.e., $D = (l_d^*, g_d^*)$, $M = (l_m^*, g_m^*)$, and $P = (l_p^*, g_p^*)$, represent the long-run symmetric equilibrium in terms of leisure time and growth rate for each of the three regimes considered.

The full list of parameter values that we use in our simulations can be found in the caption of Figure 1. Here we just briefly comment on the choice of some of them. We set $\omega = 0.1$ and $\theta = 0.9$ to represent the fact that leisure alone does not directly provide much utility as compared with relational goods. This is actually well documented by the literature on subjective well-being and
relational activities. We set $\xi = 0.9$ to represent the fact that, although leisure is an essential input, the production of relational goods mainly uses the stock of social ties as input. This is also consistent with the literature on relational goods (Gui and Sugden, 2005). Note that $\omega + \theta = 1$ which is the “weight” of material consumption in the utility function. Note also that, under these values of $\omega$, $\theta$, and $\xi$, we obtain $\phi = 0.19$ and $\eta = 0.81$. The first is a measure of the net relative weight of leisure in the utility function, while the second is a measure of the relative weight of social ties, as induced by the flow of relational goods. These values have no direct empirical support but it seems fair – if not conservative – to have material consumption weighted as much as five times leisure and at least 20% more than social ties. We set $\gamma = 0.5$ following the indication of available empirical evidence on the negative consumption externalities (see, e.g., Ferrer-i-Carbonell, 2005). We note that with these parameter values material consumption is far from useless to an individual household. On the production side, we set $\alpha = 0.75$ to obtain that, given the restriction of constant returns to scale in the accumulable factors (knowledge and physical capital), labor and knowledge can have similar elasticities in the production function. This also means that production externalities are sizable and account for a significant part of the total product. Admittedly, we are unsure about the empirical relevance of such large production externalities, but for the purpose of this paper this is not an issue. On the contrary, large positive production externalities make our point about the feasibility of happy degrowth stronger. Finally, we set $\delta = 0.01$ and $\zeta = 0.2$. This is meant to represent the substantial difference between the depreciation rate of the two stocks, and in particular the fact that social ties decay quickly if not actively sustained.

Given these parameter values, symmetric equilibria are such that $l^*_m < l^*_d < l^*_p$. As expected, the share of leisure in the decentralized economy is higher than that in the myopically planned economy. This is because the myopically government tends to increase labor supply in order to better exploit the advantages of positive production externalities. Moreover, we see that leisure is largest in the non-myopically planned economy. We stress that this is not a necessary outcome of the model, and that it emerges despite the fact that leisure is not dramatically important for current well-being. The reason for this result is that social ties are crucial for the production of relational goods, and social ties require leisure to be accumulated and maintained, so that a fully informed planner finds it convenient to raise leisure time dramatically even if this does not provide a large increase in current utility.

Furthermore, symmetric equilibria are such that $g^*_p < g^*_d < g^*_m$. As expected, the myopically planned economy has the highest rate of growth. More interestingly, the non-myopically planned economy experiences the lowest growth – less than the 1% – which, again, is not a necessary outcome of the model. Hence we see that reasonable consumption and leisure externalities, as well as the dynamics of social ties, can provide a rationale to raise leisure despite the presence of strong production externalities.

The numerical exploration that we have provided is obviously only an instance among many possible. However, we think it describes an interesting and plausible case. A natural question to ask is then how robust these results are to changes in the parameter values. The answer is: it depends. The outcome $l^*_m < l^*_d < l^*_p$ proves quite robust, as it holds for a large range of parameter values. Moreover, the relationships $g^*_p < g^*_m$ and $g^*_d < g^*_m$ always hold. However, the relation between $g^*_p$ and $g^*_d$ is definitely ambiguous, as it is very sensitive to the relative strength of the different kinds of externalities.
4.2 Happy degrowth versus unhappy growth

We now turn to the transition from a sub-optimal path (M or D) to the optimal balanced growth path $P$. Evidently, there are infinite trajectories which may take the economy from a sub-optimal path onto $P$. In particular, since the symmetric equilibrium is uniquely determined by $(l^*_m, c^*_m/k^*_m)$, the simplest way to move towards $P$ is to set immediately both $l$ and $c/k$ at their equilibrium values in the fully informed regime, so that the economy instantaneously grows at the rate $g^*_p$. However, even in this case there is a transition path due to the dynamics of social ties that has to grow until its equilibrium value is reached. Note that, since $c^*_m/k^*_m < c^*_m/k^*_m$, material consumption should discontinuously jump to a significantly lower level, which in turn produces a reduction in utility which might well not be fully offset by the extra utility granted by the sudden increase in leisure (from $l^*_m$ to $l^*_p$).

It is interesting to note that such a transition path is not optimal. The intuition is simple. Consider a corresponding model without the stock of social ties. Since production is characterized by an AK technology there would not be a proper transitional dynamics, as the level of leisure and the consumption-capital ratio would jump instantaneously to their equilibrium level. However, in a model with accumulation of social ties the increase of leisure has an additional effect with respect to the model without social ties: an increase in the future benefits of leisure due to the accumulation and maintenance of social ties. Given that the preference discount rate is positive, the choice between present and future consumption will adjust to take advantage of those future gains. In other words, the dynamics of social ties makes it convenient to smooth consumption during the transition.

Following this intuition, we can explore the dynamics of material consumption when leisure jumps to its long-run equilibrium level. Figure 2 shows the optimal transitional dynamics in the plane $(k, c)$. At the time the transition starts – i.e., time 0 – consumption has to jump and then follow its “Keynes-Ramsey” rule to approach the equilibrium level of consumption-capital ratio – i.e., the dashed line. The size in the difference between the initial and the equilibrium stock of social ties determines whether consumption in $t = 0$ has to discontinuously increase or decrease. In any case, along the trajectory towards the balanced growth path consumption growth is always smaller than in the balanced growth path. In particular, consumption can decrease sharply. Intuitively, this is due to the necessity to keep leisure high enough to accumulate $V$. To this same aim, capital may decrease for a certain period, as it happens in the depicted case. We also numerically explore the optimal transition path without imposing that leisure instantaneously jumps to its long-run equilibrium level. We find that the qualitative features of the unrestricted optimal transition path are substantially similar to the features of the transition path depicted in Figure 2.

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12 Given the complexity of the dynamical system outside the balanced growth path, it is not possible to analytically determine, along the optimal transition path, the sign of the variations of consumption and leisure. In Appendix B we provide the first order conditions and the associated “Keynes-Ramsey” rule.

13 We prove this result numerically. We compute the value function from zero to infinity when consumption and leisure jump instantaneously to their equilibrium level, and we compare this value with that obtained in a transition path where consumption and leisure first jump to an intermediate value for a while, and then move to their steady state levels. We find that the value function in the latter path is higher.

14 In Appendix B we illustrate how the dynamical system in $(k, c)$ is obtained and we prove some of its properties. In particular, we provide a necessary and sufficient condition for degrowth to take place at the beginning of transition.

15 In Appendix C, we calculate numerically optimal transition trajectories under different parameter values. We
The transition path depicted in Figure 2 may well be of happy degrowth, but it seems unlikely that a society can effectively induce (or command) a sharp and large jump in leisure time. Thus, in the following we explore the possibility for happy degrowth along a smoother transition path.

Indeed a more cautious and pragmatic strategy could be that of inducing a gradual transition from \((l^*_m, c^*_m/k^*_m)\) to \((l^*_p, c^*_p/k^*_p)\) through a smooth increase in \(l\) and a smooth reduction in \(c/k\). In figure 3 we provide numerical simulations of the relevant magnitudes for such a transition path under the parameter values of figure 1. We assume that the fully informed planner begins the transition at period \(\bar{t} = 5\) and applies a 5% growth rate of leisure. In this simulation, household leisure reaches \(l^*_p\) at time \(\bar{\bar{t}} \approx 23\). The value of \(B\) and the initial level of the capital stock \(k(0)\) only determine the scale of the stock of social ties but do not affect the basic features of the transition path. The dashed curves refer to the variables along the balanced growth path \(M\), the solid curves refer to the transition towards \(P\), and the dotted curves refer to the paths of the variables along \(P\).

Given the dynamics of leisure and capital stock along the transition path, we see that there exists only one rate of growth of material consumption such that at time \(\bar{\bar{t}}\) the ratio \(c/k\) equals the equilibrium value \(c^*_p/k^*_p\). As Figure 3.a shows, this rate is negative, \(g_{c,(m\rightarrow p)} \approx -0.5\%\) for \(\bar{t} \leq t \leq \bar{\bar{t}}\). Hence, during the transition path the economy experiences degrowth in material consumption. Once \(c/k\) reaches \(c^*_p/k^*_p\), material consumption and capital follow the trajectory described by the steady state \(P\). The degrowth of material consumption allows the accumulation of capital to go on growing for some time. However, at \(t = \bar{\bar{t}}\) the capital stock begins to decrease and continues until \(t = \bar{\bar{t}}\), that is when \(c/k\) and \(l\) attain their levels in \(P\). Hence, for \(\bar{\bar{t}} \leq t \leq \bar{\bar{t}}\) the economy follows a path characterized by degrowth in both capital and material consumption. Nevertheless, during the whole transition both the stock of social ties and household well-being grow continuously, as shown in figures 3.c and 3.d. Most importantly, the rate of growth of instantaneous utility is higher than the rate associated with the balanced growth path under a myopic social planner for most of the highlight for which intervals of admissible parameters the dynamical system initially follows a degrowth transition in consumption and/or in the stock of capital.
Figure 3: Long-run steady states with a myopic (dashed line) and fully informed planner (dotted line), and transition paths from the first to the second regime (solid line). Panel (a) shows the dynamics of household consumption, panel (b) the dynamics of household capital stock, panel (c) the dynamics of the overall stock of social ties, and panel (d) shows the dynamics of household utility. The applied parameter values are the following: $\gamma = 0.5$, $\omega = 0.1$, $\theta = 0.9$, $\sigma = 2.5$, $\xi = 0.9$, $\alpha = 0.75$, $A = 0.12$, $\delta = 0.01$, $\zeta = 0.2$, $\rho = 0.05$, $B = 10$, $K(0) = 50$. 
There are two key factors for this outcome. The first is the difference between $l_p^* - l_m^*$. Since more leisure implies less labor, the larger is $l_p^* - l_m^*$ the more likely that the economy faces a decrease in capital stock. The second factor is the difference between $c_p^* / k_p^*$ and $c_m^* / k_m^*$. Evidently, the larger is $c_m^* / k_m^* - c_p^* / k_p^*$ the more likely that material consumption must undergo a substantial reduction. In particular, since reductions in material consumption tend to sustain capital accumulation through a higher quota of savings and investment, a reduction in material consumption also dilutes the effects of a higher leisure on the accumulation of capital.

We can apply a similar reasoning to the transition from $D$ to $P$. From $l_d^* > l_m^*$ and $c_d^* / k_d^* > c_m^* / k_m^*$ we see that the two factors discussed above are, respectively, one weaker and one stronger. There is, however, a further element that pushes towards the existence of a happy degrowth in this case: the fact that the rate of growth in $D$ is smaller than the rate of growth in $M$.

5 Concluding Remarks

In this paper we investigated the relationship between growth and well-being by studying an endogenous growth model with externalities in consumption, leisure, and production. More precisely, we considered three regimes: a decentralized economy where every household takes isolated choices without considering their external effects, a planned economy where a myopic planner fails to recognize both leisure and consumption externalities but recognizes production externalities, and a planned economy where the planner is fully informed. In terms of interpretation, these three regimes can thought of as, respectively, a market economy without much Government intervention, a market economy with substantial Government intervention where however only GDP growth is recognized as relevant, and a benchmark economy where the first best is attained.

We compared the balanced growth paths under the three regimes finding that in a decentralized economy either growth or labor (or both) are too large from a welfare standpoint. Moreover, we found that a myopic intervention which internalizes production externalities but overlooks consumption and leisure externalities can lead to even greater inefficiencies in terms of too much work and consumption. Overall, we found that it could be optimal to have an average level of leisure that is dramatically higher than that emerging in any of the first two regimes. The main reason for this is that households in the first regime as well as the myopic planner in the second regime do not engage in the accumulation and maintenance of the stock of social ties.

Our results suggest that policies aiming at inducing greater leisure time can turn out to be Pareto improving. Although we did not investigate how a public authority could actually shift the economy in such a direction — since it would go far beyond the scope of the present paper — we think that limiting the intervention to an imposition of taxes on economic activity could be largely insufficient. In this regard, we believe that taxes should be coupled with policies which both increase the chance of social interactions and reduce the costs of relational activities. Needless to say, such a crucial issue deserves special attention in future research on happy degrowth.

Furthermore, we explored the transition from a sub-optimal balanced growth path to the optimal one, looking for the possibility of a degrowth transition path where welfare is increasing. Numerical simulations of transitional dynamics allowed us to demonstrate that a happy degrowth transition exists. More precisely, we showed that under a set of reasonable parameter values the transition to the first best equilibrium is associated with production downscaling, a reduction in private consumption,
and an ongoing increase in leisure and well-being. The intuition for this result is straightforward. By increasing leisure time and reducing material consumption we induce an accumulation of social ties which in turn intensifies the positive spillovers of leisure. The resulting extra utility can in turn more than compensate the utility reduction due to a smaller material consumption. We emphasize that a non-extreme dependency of individual well-being on the growth of material consumption is necessary for the political feasibility of such a transition in modern democracies, since households have to find it individually worth accepting a coordinated reduction in current material consumption. We think that negative consumption externalities arising from preferences for social status are likely to help in this regard.

A couple of final remarks are worth making. In the first place, let us stress that – at least from a purely logical standpoint – our results do not seem very surprising given the nature of the external effects that we consider. Nevertheless, we believe that our analysis provides a valuable contribution to the debate on the desirability of degrowth since the consequences of considering all such externalities together are seldom taken into account, if ever. In the second place, we stress that our results are obtained by abstracting from the negative externalities of growth on the stock of social ties and on the environment. Of course we are aware that in the real world these negative external effects may play an important role – e.g., ecological sustainability and energy shortages can undermine the desirability of economic growth. However, we stress that since such external effects obviously make economic growth less desirable, abstracting from them makes our case for happy degrowth stronger. And if one agrees that ecological limits will make a degrowth transition unavoidable – as recently argued, for instance, by Kallis (2011) – then our contribution can be seen as the proof that degrowth has the potential to be socially sustainable.

6 Acknowledgment

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References


16 For an interesting discussion on the possibility of democratic societies to accepting a reduction in consumption see Matthey (2010).

17 See for instance the classical works by Meadows et al. (1972) Daly (1973) and Georgescu-Roegen (1976). For a recent critical investigation on steady-state economy see Kerschner (2010), while for the problem of energy shortages see D’Alessandro et al. (2010) and references therein.


Appendix A: Uniqueness of balanced growth path

Decentralized economy. Substituting (11) into (9), equating the result to (10), and rearranging terms we get:

$$A(1 - l_d)^{(1 - \alpha)} - \frac{(1 - \alpha)A(1 - l_d)^{-\alpha} l_d}{\phi} = \frac{\alpha A(1 - l_d)^{(1 - \alpha)} - (\rho + \delta)}{1 - (1 - \gamma)(1 - \sigma)} + \delta$$  \hspace{1cm} (21)$$

Note that both sides of (22) are decreasing in $l_d \in [0, 1]$. Since we are interested in solutions where $l_d \in (0, 1)$ we can multiply both sides of (21) by $(1 - l_d)^\alpha$, obtaining:

$$A(1 - l_d) - \frac{(1 - \alpha)Al_d}{\phi} = \frac{\alpha A(1 - l_d) - (\rho + \delta)(1 - l_d)^\alpha}{1 - (1 - \gamma)(1 - \sigma)} + \delta(1 - l_d)^\alpha$$  \hspace{1cm} (22)$$

For $l_d \to 1$ the left-hand side is certainly lower than the right-hand side, since the former is negative and the latter is zero. Therefore, a necessary and sufficient condition to have one and just one $l_d \in (0, 1)$ such that $g_{k,d} = g_{c,d}$ is that the left-hand side of (22) is greater than the right-hand side for $l_d \to 0$. This is

$$A - \delta > \frac{\alpha A - (\rho + \delta)}{1 - (1 - \gamma)(1 - \sigma)}.$$  \hspace{1cm} (23)$$

From equation (9) we see that $A - \delta > 0$ is a necessary condition for the existence of a positive balanced growth path. Hence, inequality (23) holds for any positive balanced growth path. Thus if $g_{k,d} = g_{c,d} > 0$, then there exists one and just one balanced growth path.

Myopically planned economy. The reasoning applied for the case of a decentralized economy applies here as well, with the only difference that in the right-hand side of (14) the coefficient $\alpha$ which multiplies the first term of the numerator in (10) does not appear. For this reason the necessary and sufficient condition to have one and just one $l_m \in (0, 1)$ such that $g_{k,m} = g_{c,m} \geq 0$ is

$$A - \delta > \frac{A - (\rho + \delta)}{1 - (1 - \gamma)(1 - \sigma)}.$$  \hspace{1cm} (24)$$

As in the Decentralized Economy, $A - \delta > 0$ is a necessary condition for the existence of a positive balanced growth path. We note that such a condition implies (24). Thus also in this case we have that, if $g_{k,m} = g_{c,m} > 0$, then there exists one and just one balanced growth path.
Non-myopically planned economy. We substitute (19) into (17) and equate the result to (18). Then, since we are interested only in solutions where \( l_p \in (0, 1) \), we multiply both sides of (21) by \((1 - l_d)^{\alpha - 1}\) and rearrange terms to get:

\[
A \Psi = \frac{1}{(1 - l_p)} \left[ \frac{(1 - \gamma)(1 - \alpha) Al_p}{\eta \zeta} - \frac{(\delta + \rho)(1 - l_p)^{\alpha}}{1 - (1 - \gamma)(1 - \sigma)} \right] + \frac{\delta}{(1 - l_p)} \quad (25)
\]

Note first that the left-hand side of (25) is constant in \( l_p \). Note also the two terms in the square brackets of the right-hand side are both increasing in \( l_p \in (0, 1) \), as is the term \( \delta(1 - l_p) \). Therefore, the right-hand side of (25) is increasing in \( l_p \in (0, 1) \). Finally, note that for \( l_p \to 1 \) the right-hand side of (25) goes to infinity, so that a necessary and sufficient condition to have one and just one \( l_p \in (0, 1) \) such that \( g_{k,p} = g_{c,p} \) is that the left-hand side of (25) is smaller than the right-hand side for \( l_p \to 0 \). We note that this is the case if and only if (24) holds.

The latter equivalence, together with fact that in this case too \( A - \delta > 0 \) is a necessary condition for the existence of a positive balanced growth path, imply that if \( g_{k,m} = g_{c,m} > 0 \) then there exists one and just one balanced growth path.
Appendix B: Optimal growth and transitional dynamics

Balanced growth path. The current value of Hamiltonian \( H \) for the economy with fully informed planner is given by the following expression (the time variable is omitted for simplicity):

\[
H = \frac{(c^{1-\gamma}l^{\phi}V^{\eta})^{1-\sigma}}{1-\sigma} + \lambda [Ak(1-l)^{1-\alpha} - \delta k - c] + \mu(Bl - \zeta V),
\]

where \( \lambda \) and \( \mu \) are the co-state variables for \( k \) and \( V \) respectively. Since \( c \) and \( l \) are choice variables, the first order conditions yield:

\[
\frac{(1-\gamma)}{c}(c^{1-\gamma}l^{\phi}V^{\eta})^{1-\sigma} = \lambda,
\]

\[
\frac{\phi}{l}(c^{1-\gamma}l^{\phi}V^{\eta})^{1-\sigma} = \lambda(1-\alpha)Ak(1-l)^{-\alpha} - \mu B,
\]

\[
\dot{\lambda} = \lambda \left[ \rho + \delta - A(1-l)^{1-\alpha} \right],
\]

\[
\dot{\mu} = \left( \rho + \zeta \right) \mu - \frac{\eta(c^{1-\gamma}l^{\phi}V^{\eta})^{1-\sigma}}{V},
\]

plus the two constraints (12) and (5).

Equations (27)–(30), (12) and (5) together with the non-negative constraints and the transversality conditions for the two stocks, give us the necessary and sufficient conditions for the optimal path. By taking the time derivative of (27) and from equation (29) we obtain the “Keynes-Ramsey” rule, that is

\[
\frac{\dot{c}}{c} = \frac{1}{1 - (1-\gamma)(1-\sigma)} \left[ A(1-l)^{1-\alpha} - \rho - \delta + \phi(1-\sigma)\frac{l}{l} + \eta(1-\sigma)\frac{V}{V} \right].
\]

Since along the balanced growth path the growth rate of leisure and social ties must be equal to zero, from equations (29) and (31), we obtain equation (18). Furthermore, from equations (27) and (28), we obtain

\[
\mu = \frac{(c^{1-\gamma}l^{\phi}V^{\eta})^{1-\sigma}}{lB} \left[ (1-\gamma)(1-\alpha)Ak(1-l)^{-\alpha}l - \phi \right].
\]

Along the balanced growth path, by taking the time derivative of (32), and from equations (5) and (32) we obtain equation (19).

Transitional dynamics. Equation (31) can be used to analyze the transitional dynamics towards the balanced growth induced by the fully informed planner when leisure discontinuously jumps to the new equilibrium level. Given \( l(t) = l^*_p \), equation (5) can be solved to obtain the path of social ties accumulation:

\[
V(t) = \left( V(0) - \frac{Bl^*_p}{\zeta} \right) e^{-\xi t} + \frac{Bl^*_p}{\zeta}.
\]
where \( V(0) \) is stock of social ties which exists at the beginning of the transition. From \( l(t) = l^*_p \) and equations (31) and (33), we obtain

\[
\frac{\dot{c}}{c} = \frac{A(1 - l^*_p)^{1-\alpha} - \rho - \delta + \eta (1 - \sigma) \left[ \frac{B l^*_m}{(V(0) - \frac{B l^*_m}{\zeta})} e^{-\zeta t} + \frac{B l^*_m}{\zeta} \right] - \zeta}{1 - (1 - \gamma)(1 - \sigma)}. \tag{34}
\]

Exploiting equations (38) and (12) the dynamics of consumption and capital can be jointly analyzed.

Figure 2 in the paper shows the unique trajectory \( c(t) = f(k(t)) \) which converges to the balanced growth path.

Equation (38) can be solved. To fix ideas, let the transition start in \( t = 0 \) from the balanced growth path of the myopically planned economy. In particular, we have that \( V(0) = \frac{B l^*_m}{\zeta} \). Thus, the solution of (38) is

\[
c(t) = e^{\frac{\zeta (1 - \sigma)(l^*_m - l^*_p)}{1 - \psi}} \left[ e^{-\zeta t}(l^*_m - l^*_p) + l^*_p \right] \frac{\eta (\sigma - 1)}{1 - \psi} D; \tag{35}
\]

where \( D = c(0) l^*_m \frac{\eta (\sigma - 1)}{1 - \psi} \) is the constant obtained in \( t = 0 \). Then, we can compute rate of growth of (35):

\[
\frac{\dot{c}(t)}{c(t)} = \frac{A(1 - l^*_p)^{1-\alpha} - \rho - \delta}{1 - \psi} + \frac{\eta (1 - \sigma) (l^*_m - l^*_p)}{(l^*_m - l^*_p + l^*_s e^{\zeta t})(1 - \psi)}. \tag{36}
\]

As one can see, the first term in (36) is the rate of growth of the balanced growth path for the economy with a fully informed planner, while the second term is always negative – since \( \sigma > 1 \), \( \zeta > 0 \), \( \eta > 0 \), and \( \psi < 0 \) – and tends to 0 as \( t \) tends to infinity. Hence, we have that \( \dot{c}(t)/c(t) \) tends to \( g_{c,p} \) from below as \( t \) tends to infinity. In other words, along the trajectory towards the new balanced growth path consumption growth is always smaller than in the balanced growth path. Intuitively, this is due to the necessity to keep leisure high enough to accumulate \( V \).

Moreover, it can be proved that material consumption starts decreasing in \( t = 0 \) if and only if

\[
g_{c,p} \leq \frac{\zeta \eta (\sigma - 1) l^*_p - l^*_m}{1 - \psi} l^*_m. \tag{37}
\]

If (37) holds then the trajectory towards the new balanced growth path is characterized by a period of degrowth. How long is such a degrowth period depends on the model parameters. More precisely, we have that degrowth happens up to time:

\[
\bar{t} = \frac{1}{\zeta} \left\{ \ln \left[ \frac{\zeta \eta (\sigma - 1)(l^*_p - l^*_m)}{A(1 - l^*_p)^{1-\alpha} - \rho - \delta + (l^*_m - l^*_p)} \right] - \ln(l^*_p) \right\}. \tag{38}
\]
Appendix C: Robustness analysis

In order to explore the robustness of the case for happy degrowth presented in Section 4, to which we refer as the baseline case, we calculate numerically optimal trajectories under different parameter values. As for the baseline case, we focus on trajectories for a transition from the balanced growth path of the myopically planned economy to the balanced growth path of the non-myopically planned economy.

With respect to the baseline case, we change in turn one and only one parameter. This means that each trajectory is obtained for a set of parameter values that differs from the set used for the baseline case just for one parameter value. In other words, this is a one-dimensional exploration of robustness.

Results are summarized in table 1. Numerical simulations show that an initial degrowth in material consumption – that characterized the baseline case – obtains for wide range of parameter values. Initial degrowth of the stock of physical capital is obtained for a smaller range of parameter values, but the range is far from being negligible. The absence of degrowth in both material consumption and the stock of capital obtains for a rather small range of parameter values. We think that these results suggest that the case for degrowth that we present in the paper is robust, in the sense that there exists a non-negligible interval of parameter values for which it obtains.

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<tr>
<th>Parameter</th>
<th>Baseline</th>
<th>Range $c(t) \downarrow$, $k(t) \downarrow$</th>
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</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.75</td>
<td>0.5 – 0.85</td>
<td>0.9 – 0.95</td>
<td>–</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.5</td>
<td>0.1 – 0.7</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.9</td>
<td>0.45 – 0.9</td>
<td>0.15 – 0.45</td>
<td>0.1 – 0.15</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.1</td>
<td>0.1 – 0.4</td>
<td>0.4 – 0.9</td>
<td>–</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.9</td>
<td>0.65 – 0.9</td>
<td>0.2 – 0.6</td>
<td>0.1 – 0.2</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.05</td>
<td>0.02 – 0.05</td>
<td>0.01 – 0.015</td>
<td>–</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.01</td>
<td>0.005 – 0.025</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.2</td>
<td>0.04 – 0.2</td>
<td>0.02 – 0.035</td>
<td>0.01 – 0.015</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2.5</td>
<td>1.3 – 5</td>
<td>1.1 – 1.25</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 1: Simulated trajectories for a transition from the balanced growth path of the myopically planned economy to the balanced growth path of the non-myopically planned economy. Simulations are carried out for sets of parameter values which differs from the baseline set of parameters for one parameter at time. Second column indicates which parameter is let vary. Third column shows the parameter range where both consumption and the stock of physical capital initially decrease. Fourth column shows the parameter range where consumption initially decreases while and the stock of physical capital initially increases. Fifth column shows the parameter range where both consumption and the stock of physical capital initially increase.
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