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with strict single crossing  
in types and actions**

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# Strict Nash equilibria in large games with strict single crossing in types and actions

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## Abstract

In this paper we study games where the space of player types is atomless, action spaces are second countable, and payoffs functions satisfy the property of strict single crossing in types and actions. Our main finding is that in this class of games every Nash equilibrium is essentially strict. We briefly develop and discuss the relevant consequences of our result.

*Key words:* atomless; single crossing; strict Nash; pure Nash; monotone Nash

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## 1. Introduction

Strict Nash equilibria are important in game theory for many reasons. One of the most prominent is that they are immune from an annoying critique which is often directed towards weak Nash equilibria: why should a player choose precisely her equilibrium action among the ones she is indifferent to? Another reason why strict equilibria are important is that they possess relevant stability properties such as evolutionary stability (see e.g. Crawford, 1990) and asymptotic stability (see e.g. Ritzberger and Weibull, 1995). Last but not least, if an equilibrium is strict then it is also pure, and this is regarded as a desirable property (see e.g. Morris, 2008). In this paper we prove that in games with an atomless space of player types and payoff functions satisfying the property of strict single crossing in types and actions, all Nash equilibria are essentially strict. We emphasize that we consider action sets of arbitrary cardinality with the only topological requirement of being second countable.<sup>1</sup>

The assumption of an atomless space of player types is typical of the literature on the existence of pure equilibria in large games (see Khan and Sun, 2002, for an exhaustive survey on large games). A first important result in this stream is provided by Schmeidler (1973) for the class of games in which each player's payoff depends only on her choice and on the average choice of the others. Schmeidler's main theorem states that if the player set is endowed with an atomless Lebesgue measure then a pure Nash

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<sup>1</sup>Actually, the assumption of second countability implies a cardinality less than or equal to the cardinality of the continuum.

equilibrium exists.<sup>2,3</sup> Existence of pure Nash equilibria does not extend, however, to general games: in order to guarantee the existence of a pure strategy equilibrium either the action space or the family of payoff functions must be assumed to be countable. For this reason alternative formalizations have been explored (see [Carmona and Podczeck, 2009](#), for a recent analysis of the relationship between different formalizations).

The celebrated purification theorem in [Harsanyi \(1973\)](#) deserves a special mention in our discussion, because it has provided an important result on strict Nash equilibria.<sup>4</sup> In short, Harsanyi demonstrated that independently perturbing the payoffs of a finite normal form game produces a new game with a continuum of types where all equilibria are pure and essentially strict. Harsanyi's idea is to consider complete information games as abstract approximations of incomplete information games where each player has a private information about a small perturbation of his own payoff function.

However, Harsanyi's purification theorem goes well beyond this. It also states that for any regular equilibrium of the original game and any sequence of the perturbed games converging to the original one, there is a sequence of essentially strict and essentially pure equilibria converging to the regular equilibrium. This latter result has the notable merit of having reassured most applied economists that mixed strategy equilibria of the unperturbed game can be used in a meaningful way. However, the price for this laudable result has been to restrict attention to regular equilibria.

A series of contributions extended the work of [Harsanyi \(1973\)](#) providing more general conditions for the existence of pure equilibria, but disregarding the issue of approachability (see [Morris, 2008](#), and references therein). The basic question asked in these papers is when mixed strategy equilibria can be replaced by (perhaps approximately) equivalent pure strategy equilibria (see e.g. [Radner and Rosenthal, 1982](#); [Milgrom and Weber, 1985](#); [Aumann et al., 1983](#)).<sup>5</sup> This stream of research produced interesting variants of purification results. However, conditions granting strict equilibria were no longer investigated.

In our analysis we disregard the issue of approachability as well but we do focus on strict Nash equilibria. Moreover, we stress that we employ second countable action spaces of arbitrary cardinality coupled with an uncountable set of player types. The price for our result is paid in terms of a loss of generality on payoff functions: we restrict to games where the payoff functions satisfy the strict single crossing property ([Milgrom and Shannon, 1994](#)) in player types and actions. We are aware that this is a severe restriction. However, we think that such an assumption is less harsh when we come to applied models, where instead the possibility to work with action spaces such as the real line (or its intervals) is usually appreciated.

The use of single crossing properties is not new in the literature on games with many players (or, following a different interpretation, with many types). [Athey \(2001\)](#) analyzes games of incomplete information where each agent has private information about her own type, and the types are drawn from an atomless joint probability distribution. The main result establishes existence of pure Nash equilibria under an assumption called single crossing condition for games of incomplete information, which is a weak version of the single crossing property in [Milgrom and Shannon \(1994\)](#).<sup>6</sup> In the last section we argue that such a property is not a sensible generalization for our purposes.

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<sup>2</sup>[Mas-Colell \(1984\)](#) deals with the issue of [Schmeidler \(1973\)](#) using a different approach based on distributions rather than measurable functions. He is able to obtain a pure strategy existence result without the atomless assumption, at the price of letting players with identical utility function free to choose different actions. Moreover, he allows for more general dependence of utility functions on societal responses than the average choice of the others.

<sup>3</sup>Approximated versions of the result in [Schmeidler \(1973\)](#) have been given for a large but finite number of players ([Rashid, 1983](#); [Carmona, 2004, 2008](#)).

<sup>4</sup>See [Govindan et al. \(2003\)](#) for a slight extension of Harsanyi's main theorem and an alternative simpler proof.

<sup>5</sup>These purification results build on the interpretation of mixed strategies first suggested by ([Aumann, 1974](#)): randomization is not deliberate but represents in fact uncertainty in the players' mind about how the opponents will play.

<sup>6</sup>[Reny and Zamir \(2004\)](#) prove existence of pure strategy Nash equilibria under a slightly weaker condition. [MacAdams \(2003\)](#)

The paper is organized as follows. In Section 2 we introduce the assumptions. In Section 3 we state our main result: every Nash equilibrium is essentially strict. Moreover, we provide a straightforward corollary where we get that all Nash equilibria are essentially monotone, meaning that equilibrium actions are increasing – but not strictly increasing – in player types.

Finally, in Section 4 we present an articulated discussion of our findings. First, we point out that our result implies that every Nash equilibrium is essentially pure in the class of games that we consider. Second, we remark that equilibrium existence is not ensured in the general framework we deal with; however, our result can easily be coupled with existence theorems of (even mixed) Nash equilibria to get existence of strict Nash equilibria. Lastly, we discuss the assumptions under which we derived our main result, arguing in favor of their tightness.

## 2. Assumptions

We consider a set  $T$  of player types which is partitioned into  $\{T_i\}_{i \in I}$  subsets, with  $I$  a countable set whose elements are interpreted as player roles. For short, we will refer to the elements of  $T$  as types and to the elements of  $I$  as roles.<sup>7</sup> Every  $T_i$  is totally ordered by a relation  $\leq_i^T$ . We also assume that  $(T, \mathcal{T}, \tau)$  is an atomless probability space where  $\{t\} \in \mathcal{T}$  for every  $t \in T$ .

For all  $t \in T$ , let  $A_t$  denote the set of actions available to type  $t$ . We assume that actions are equal within roles, namely if  $t, t' \in T_i$ , then  $A_t = A_{t'}$ . We use  $A_i$ , for  $i \in I$  when we do not refer to a distinctive type but to all types in role  $i$ . Every  $A_i$  is totally ordered by a relation  $\leq_i^A$ . Every  $A_i$  is assumed to be second countable<sup>8</sup> with respect to the order topology induced by  $\leq_i^A$ . We denote  $\prod_{t \in T} A_t$  with  $F$  and  $\prod_{t' \in T, t' \neq t} A_{t'}$  with  $F_{-t}$ . An element  $f \in F$  is called a profile of actions while  $f_{-t}$  denotes the restriction of  $f$  to  $F_{-t}$  and is called a profile of opponents' actions. A utility function  $u : T \times F \rightarrow \mathbb{R}$  maps for every type a profile of actions into a real number.

We now provide the formal definition of the property of strict single crossing in types and actions, which is the crucial assumption for our results.

*Strict single crossing property in types and actions.* We assume that for all  $f \in F$ ,  $i \in I$ ,  $t <_i^T t'$ , and  $a <_i^A a'$ :<sup>9</sup>

$$u(t, a', f_{-t}) \geq u(t, a, f_{-t}) \text{ implies } u(t', a', f_{-t'}) > u(t', a, f_{-t'}).$$

Finally we introduce some further definitions. A profile of actions  $f \in F$  is a *Nash equilibrium* if, for all  $t \in T$ ,  $u(t, f_t, f_{-t}) \geq u(t, a, f_{-t})$  for all  $a \in A_t$ . A profile of actions  $f$  is said to be *essentially strict* if  $\tau(\{t \in T : u(t, f_t, f_{-t}) > u(t, a, f_{-t}) \text{ for all } a \in A_t\}) = 1$ . A profile of actions  $f$  is said to be *essentially monotone* if there exists  $S \subseteq T$  such that  $\tau(S) = 1$  and for all  $t, t' \in S$ ,  $t' >_i^T t$  implies  $f_{t'} \geq_i^A f_t$ .

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further extends the analysis to multidimensional action spaces while [Reny \(2009\)](#) extends it to more general partially ordered action spaces.

<sup>7</sup>Different interpretations lead to prefer different choices of labels. The literature on large games would prefer the use of players and roles (or groups) for  $T$  and  $I$  respectively, while the literature on incomplete information games would prefer the use of types and players. We do not have major reasons to favor one interpretation over the other, and our choice of labels is somehow compatible with both.

<sup>8</sup>A topological space is said to be second countable if its topology has a countable base.

<sup>9</sup>Our definition is slightly different from the standard one in that the profile of opponents' actions, which is a third argument of function  $u$  in addition to own type and action, may vary from  $f_{-t}$  to  $f_{-t'}$ . Such a slight difference disappear if, for instance, we assume individual negligibility (see discussion in Section 4) or we constrain types to care only about actions of roles different from theirs (as happens e.g. in incomplete information games).

### 3. Essentially strict Nash equilibria

We are ready to state our main result.

**Theorem 1.** *If  $f \in F$  is a Nash equilibrium, then  $f$  is essentially strict.*

*Proof.* We define  $R_t(f)$  as the set of best replies to  $f$  for type  $t$ , namely  $R_t(f) = \{a \in A_t : u(t, a, f_{-t}) \geq u(t, a', f_{-t}) \text{ for all } a' \in A_t\}$ . We show below that for all  $i \in I$ , the set  $\{t \in T_i : \|R_t(f)\| > 1\}$  is countable. We note that for all  $i \in I$ , the set  $\{t \in T_i : \|R_t(f)\| > 1\}$  is also measurable since by assumption  $\{t\} \in \mathcal{T}$  for every  $t \in T$ . Hence, the set  $\{t \in T : \|R_t(f)\| > 1\}$  is both countable and measurable, since it is union of countably many countable and measurable sets. In particular, its measure is equal to zero because  $(T, \mathcal{T}, \tau)$  is an atomless probability space. This also implies, since  $R_t(f)$  cannot be empty when  $f$  is a Nash equilibrium, that the complement set has measure one, i.e.  $\tau(\{t \in T : \|R_t(f)\| = 1\}) = 1$ .

*Proof that  $\{t \in T_i : R_t(f) > 1\}$  is countable.* We take a function  $g : \{t \in T_i : R_t(f) > 1\} \rightarrow A_i^2$  such that  $g(t) = (g_0(t), g_1(t))$  with  $g_0(t), g_1(t) \in R_t(f)$  and  $g_0(t) \prec_i^A g_1(t)$ .

We take a countable base  $\mathcal{B}$  for the topology of  $A_i$ . We take a function  $h : g(\{t \in T_i : R_t(f) > 1\}) \rightarrow \mathcal{B}$  such that  $a_0 \in h(a_0, a_1)$  and  $a_1 \notin h(a_0, a_1)$ . To see that such a function  $h$  exists, for all  $(a_0, a_1) \in g(\{t \in T_i : R_t(f) > 1\})$  consider the open set  $\{a \in A_i : a \prec_i^A a_1\}$ ; since  $\mathcal{B}$  is a base, there must exist  $B \in \mathcal{B}$  such that  $a_0 \in B$  and  $a_1 \notin B$ .

We check that  $g$  is injective. For all  $t, t' \in \{t \in T_i : R_t(f) > 1\}$ ,  $t < t'$ , we have that  $g_0(t) < g_1(t) \leq g_0(t') < g_1(t')$  by the strict single crossing property in types and actions.

We check that  $h$  is injective. For all  $(a_0, a_1), (a'_0, a'_1) \in g(\{t \in T_i : R_t(f) > 1\})$ ,  $(a_0, a_1) \neq (a'_0, a'_1)$ , we know that either  $a_0 < a_1 \leq a'_0 < a'_1$  or  $a'_0 < a'_1 \leq a_0 < a_1$ . Hence,  $h(a_0, a_1) \neq h(a'_0, a'_1)$ .

Therefore,  $g \circ h$  maps injectively  $\{t \in T_i : R_t(f) > 1\}$  into  $\mathcal{B}$ . Since  $\mathcal{B}$  is countable, so must be  $\{t \in T_i : R_t(f) > 1\}$ . □

That  $f$  is essentially strict follows from  $(T, \mathcal{T}, \tau)$  being atomless and from the set of weakly best responders being countable. The main part of the proof is devoted to show that the set of weakly best responders is indeed countable. The crucial economic assumption is the strict single crossing property in types and actions, which implies that the sets of weakly best replies of any two distinct types intersect at most at an extreme point, and hence are – roughly speaking – rather separated one from the other. The technical assumption of second countability completes the job, allowing at most a countable number of such sets (see Section 4 for a discussion on the importance of second countability).

A straightforward application of the property of strict single crossing in types and actions yields the following corollary.

#### Corollary 1

If  $f \in F$  is a Nash equilibrium, then it is essentially monotone.

*Proof.* By Theorem 1,  $\tau(\{t \in T : \|R_t\| = 1\}) = 1$ . By strict single crossing in types and actions, for all  $t, t' \in \{t \in T : \|R_t\| = 1\}$ ,  $t' >_i^T t$  implies  $f_{t'} \succeq_i^A f_t$ . □

### 4. Discussion

Strict Nash equilibrium is a particularly appealing solution concept that has been so far given relatively little attention. Unfortunately, strict Nash equilibria do not obtain in general settings. As we have shown in this paper, however, there exists a quite general class of games for which all Nash equilibria are essentially

strict. In particular, we have considered games with an atomless space of player types, second countable action spaces, and strict single crossing property in types and actions. We think that these class of games can prove itself to be useful in various economic applications. We hope that future research on this topic might identify different and potentially more general classes of games where strict Nash equilibria naturally arise. Some remarks follow.

*Inexistence of mixed equilibria.* Our result has the almost straightforward implication that, in the extension of the game to mixed actions, non-degenerate mixed equilibria do not essentially exist. To illustrate why, we disregard some technical difficulties concerning the manufacture of appropriate probability measures and the extension of utility functions, limiting ourselves to the following observation. Since almost all types have a single best reply, almost all types optimally assign probability one to that single best reply. This essentially rules out the possibility to have mixed equilibria.

*Equilibrium existence.* We have shown that every Nash equilibrium must be essentially strict, but we have not proved that a Nash equilibrium exists. Actually, our main statement may trivially hold because no Nash equilibrium exists. To obtain existence of a strict Nash, however, it suffices to assume what is necessary for applying one of the many existence theorems – either in pure strategies or mixed strategies – that the literature provides. Among the most general we find Theorem 1 and Corollary 1 in [Khan and Sun \(2002\)](#).

*Individual negligibility.* We have derived our results under the assumption that utility depends on the action of every single type  $t \in T$ . We did this in order to state our findings in a setting which allows for a general form of utility dependency. However, we note that under the assumption of an atomless space of types – which implies the existence of uncountably many types – it may be reasonable to impose that any single type  $j \neq t$  is negligible in terms of  $t$ 's utility. This assumption is particularly reasonable if one also assumes continuity of the utility function which easily leads to negligibility (see the discussion in [Khan and Sun, 2002](#), Section 2). Such an idea of negligibility is often present in studies on the equilibria of games with a large number of players. Many contributions on this issue – inspired by the seminal paper by [Schmeidler \(1973\)](#) – have modeled negligibility by assuming that utility depends on a measure of opponents' actions. To introduce negligibility in our framework it suffices to impose that the utility function  $u$  is such that whenever  $f, f' \in F$  agree on a set of measure one according to  $\tau$ , we have that  $u(t, f) = u(t, f')$  for every  $t \in T$  such that  $f_t = f'_t$ .

*Second countability vs. separability.* A space is called separable if it contains a countable dense subset. Separability is a topological property which is weaker than second countability but plays a similar role: it constraints the topological size of the space.

However, if we assume that the action sets are separable instead of second countable, then our results fail. The following example, which is a modification of a standard argument to illustrate that a separable space need not be second countable, shows that if we replace second countability with separability then there may exist non-strict Nash equilibria. We consider a unique role, and we let the set of player types  $T$  be the real line, denoted with  $\mathbb{R}$ . We let the action set  $\mathcal{A}$  be the Cartesian product  $\mathbb{R} \times \{0, 1\}$ . We give  $\mathcal{A}$  the lexicographic order, i.e.,  $(r, i) < (s, j)$  if either  $r < s$  or else  $r = s$  and  $i < j$ . For every profile of actions  $f$ , type  $t$ 's utility function is  $u(t, f) = -(t - f'_t)^2$ , where  $f'_t = s$  if  $f_t = (s, i)$ . In the order topology  $\mathcal{A}$  is separable: the set of all points  $(q, 0)$  with  $q$  rational is a countable dense set. However,  $f$  such that  $f_t = (t, 0)$  for all types  $t$  is a non-strict Nash equilibrium where every agent  $t$  is indifferent between  $(t, 0)$  and  $(t, 1)$ .

*Strict single crossing vs. single crossing.* If the property of strict single crossing is weakened to the property of single crossing, then our results cease to hold. A straightforward counterexample is as follows. Assume that every agent has a constant utility function, so that everyone is always indifferent between any of her actions. Single crossing property is satisfied, and whatever profile of actions is a weak Nash equilibrium. This trivial example also shows that we cannot recover our main result even if we replace the property of single crossing with the stronger one of increasing difference – i.e., for all  $f \in F$ ,  $i \in I$ ,  $t' >_i^T t$  and  $a' >_i^A a$ , we have that  $u(t, a', f_{-i}) - u(t, a, f_{-i}) \leq u(t', a', f_{-i'}) - u(t', a, f_{-i'})$ .<sup>10</sup>

We note that the property used in [Athey \(2001\)](#), called single crossing condition for games of incomplete information (SCC), is a weaker version of the property of single crossing. Roughly speaking, SCC requires that the property of single crossing in types and actions holds for each (player) role when for all other roles higher types adopt higher actions. While SCC turns out to be a clever generalization of single crossing when the purpose is to prove the existence of pure Nash, such a kind of generalization applied to strict single crossing does not work when we want to show that every Nash equilibrium is essentially strict. Indeed, our main result does not hold if we replace the property of single crossing with strict SCC. The reason is that under strict SCC it remains possible that some weak Nash equilibrium exists for a profile of actions for which the property of strict single crossing does not hold.

*Monotonicity vs. strict monotonicity.* One may wonder whether [Corollary 1](#) can be refined to prove strict monotonicity instead of monotonicity. It turns out that this is not the case, even if we adopt the stronger property of strict increasing differences in types and actions – i.e., for all  $f \in F$ ,  $i \in I$ ,  $t' >_i^T t$  and  $a' >_i^A a$ , we have that  $u(t, a', f_{-i}) - u(t, a, f_{-i}) < u(t', a', f_{-i'}) - u(t', a, f_{-i'})$  – instead of strict single crossing. The following example illustrates why. Let  $\|I\| = 1$  and let both the set of types  $T$  and the set of actions  $\mathcal{A}$  be equal to the real segment  $[0, 1]$ . For every profile of actions  $f$ , type  $t$ 's utility function is  $u(t, f) = (1 + t)f$ . It is clear that there exists a unique Nash equilibrium where all types play action 1. Hence, monotonicity holds but strict monotonicity does not.

*Other-dependent order relations.* In the discussion above we have argued that our results cannot be generalized by relaxing certain assumptions. However, there is at least one generalization that is possible: the total orders for each  $T_i$  and  $A_i$  can be made dependent on the action profiles of (player) roles other than  $i$ . More precisely, if we denote with  $f_{-i}$  a profile of actions for any type other than the types of  $i$ , and with  $F_{-i}$  the set of all  $f_{-i}$ , we may assume that for all  $i \in I$ , for all  $f_{-i} \in F_{-i}$ , there exist  $\leq_i^A(f_{-i})$  and  $\leq_i^T(f_{-i})$  with respect to which the strict single crossing property in types and actions holds, i.e., for all  $t <_i^T(f_{-i}) t'$ , and  $a <_i^A(f_{-i}) a'$ ,  $u(t, a', f_{-i}) \geq u(t, a, f_{-i})$  implies  $u(t', a', f_{-i'}) > u(t', a, f_{-i'})$ . It can be easily checked that, under this weaker requirement, the proofs of our statements would remain essentially unchanged, so that the basic results in [Theorem 1](#) and [Corollary 1](#) would still be valid.

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<sup>10</sup>In this definition and following ones a remark similar to that in [footnote 9](#) applies.

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