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VAR Information and the Empirical Validation of DSGE Models

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VAR Information and the Empirical Validation of DSGE Models

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Abstract

A shock of interest can be recovered, either exactly or with a good approximation, by means of standard VAR techniques even when the structural MA representation is non-invertible or non-fundamental. We propose a measure of how informative a VAR model is for a specific shock of interest. We show how to use such a measure for the validation of shocks' transmission mechanism of DSGE models through VARs. In an application, we validate a theory of news shocks. The theory does remarkably well for all variables, but underestimates the long-run effects of technology news on TFP.

JEL classification: C32, E32.

Keywords: invertibility, non-fundamentalness, news shocks, DSGE model validation, structural VAR.

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1 Introduction

Any theoretical model should in principle be validated by evaluating whether its implications are consistent with empirical facts.

DSGE models represent the most popular class of models in theoretical macroeconomics. They are very useful to study the propagation mechanisms of economic shocks and address policy-relevant questions. However, like any theoretical model, they often rely on arbitrary assumptions, which are hard to judge from an empirical point of view. Maximum likelihood or Bayesian estimation methods provide the best constrained fit of the data; however, it might still be the case that the theory fails in fitting the data satisfactorily (Sala, 2015).

On the other hand, structural VAR models have long been the main tool in applied macroeconomic research. Structural VARs are, to a large extent, free of restrictions derived from economic theory. It is therefore quite natural to use VAR models for the empirical validation of DSGE models, along the route pioneered by Sims (1989). A prominent example of the use of VAR models for validation purposes is represented by the technology-hours debate, where the empirical response of hours to technology shocks has been used to assess RBC and sticky prices models (Gali, 1999; Christiano, Eichenbaum and Vigfusson, 2004). The topic is extensively discussed in Canova (2002, 2007), Christiano, Eichenbaum and Vigfusson (2007), Chari, Keohe and McGrattan (2008) and Giacomini (2013).

Validation through VARs is typically carried out by comparing the unrestricted VAR impulse response functions – let us call them the “empirical” impulse response functions – with the constrained ones stemming from the DSGE model – let us call them the “theoretical” impulse response functions. A crucial problem with this procedure is the following: does the VAR specification employed convey enough information to estimate the shocks of interest and the related response functions, under the null hypothesis that the model is true? If this is not the case, the VAR and the DSGE model are incompatible and the comparison is inconclusive, whatever the outcome might be. The key question is therefore: how can we know whether a VAR specification is deficient, according to the theoretical model?

In this paper we propose a measure of the informational deficiency of a given VAR specification with respect to any particular shock. This measure, which we call $\delta_i$, tells us the best that we can do in approximating the shock of interest $u_{it}$ by means of a VAR. The measure is defined as the fraction of unexplained variance of the orthogonal projection of $u_{it}$ onto the VAR residuals. It can be computed for any DSGE model, endowed with a set of values for the parameters. It takes on values between zero and one; $\delta_i = 0$ means perfect information for the shock $u_{it}$; $\delta_i = 1$ means no information. If $\delta_i = 0$ we say that the VAR is informationally sufficient for $u_{it}$. If $\delta_i$ is close to zero, the VAR is approximately sufficient. A VAR which is
sufficient, or approximately sufficient, can be used for model validation.

To better clarify the practical use of our measure, let us explain, step by step, the validation procedure we have in mind. First, consider the log-linear equilibrium representation of a calibrated/estimated DSGE model. Second, compute the measure for a particular VAR specification (whose variables form a subset of the variables modeled in the DSGE). For the sake of simplicity, let us focus on just a single shock $u_{it}$. Then use the following “$\delta$-criterion”. If $\delta_i$ is larger than a pre-specified threshold level, e.g. 0.05, reject the specification and choose another vector of observables. If it is smaller, estimate the VAR (with real data) and identify the shocks of interest using restrictions consistent with the theoretical model. Finally, verify whether the theoretical impulse response functions lie within the confidence bands obtained with the VAR. If they do, the model is validated. If they do not, there is something wrong with either the parameter calibration/estimation or the model itself.

The concept of “sufficient information” used here is due to Forni and Gambetti (2014). It is a generalization of the concept of “invertibility”, or “fundamentalness” (Lippi and Reichlin, 1993), which has been widely debated in the recent literature.\footnote{Early papers are Hansen and Sargent (1991) and Lippi and Reichlin (1993, 1994b). A partial list of recent papers includes Giannone, Reichlin and Sala (2006), Giannone and Reichlin (2006), Fernandez-Villaverde, Rubio-Ramirez, Sargent and Watson (2007), Ravenna (2007), Yang (2008), Forni, Giannone, Lippi and Reichlin (2009), Mertens and Ravn (2010), Sims (2012), Leeper, Walker and Yang (2013), Forni, Gambetti, Lippi and Sala (2013a, 2013b), Forni, Gambetti and Sala (2014), Forni and Gambetti (2014), Beaudry and Portier (2015).} Precisely, we have fundamentalness if and only if the VAR is informationally sufficient for all of the structural shocks. Correspondingly, the deficiency measure proposed in the present paper can be regarded as a generalization of existing fundamentalness conditions, including the well-known “Poor Man’s Condition” of Fernandez-Villaverde et al. (2007) (PMC hereafter). The generalization works in two dimensions: (a) our measure is shock-specific; (b) it provides information about the “degree” of nonfundamentalness.

Why is this generalization important? The reason is that fundamentalness is extremely and unnecessarily restrictive.

In several cases fundamentalness is not needed. The researcher is often interested in assessing the transmission mechanisms of a single shock. We show below that a VAR may be informationally sufficient for this particular shock, even if it is not sufficient for all shocks, i.e. fundamentalness does not hold. Moreover, VAR deficiency, though different from zero, may be very small; in this case, the VAR performs reasonably well despite nonfundamentalness. Indeed, what is really important for a reliable VAR analysis is not whether we have fundamentalness or not, but whether we have enough information to get a reasonably good approximation for the specific shocks of interest.

Moreover, fundamentalness is very demanding. If the number of variables in the VAR
is smaller than the number of shocks in the DSGE, fundamentalness cannot hold (and the
PMC is not even defined). Since modern DSGE models have several shocks, small VARs are
automatically ruled out. This is unpleasant, since small VARs are useful tools, especially for
short samples. In addition, if the number of variables in the VAR is equal to the number of
structural shocks, the existence of a specification satisfying the PMC is far from being assured.
For instance, under fiscal foresight, or in presence of news shocks, nonfundamentalness is
endemic (Leeper, Walker and Yang, 2013; Sims, 2012). By using the $\delta$-criterion, in place
of existing fundamentalness conditions, we give small VARs a fair chance and substantially
increase the probability to find a suitable specification.

The fact that a VAR might perform well despite nonfundamentalness was first observed by
Sims (2012) and further documented in Beaudry and Portier (2015). In this paper we show
that the performance of a VAR in recovering $u_{it}$ and the related impulse response functions is
closely related to our deficiency measure. Our work is also related to Soccorsi (2015), where
a fundamentalness measure is proposed. The main difference with respect to ours is that it is
global, rather than shock-specific, and, like PMC, it is only defined for square systems, having
as many variables as shocks.

Nonfundamentalness is not the only problem for VAR validation of DSGE models. Two
additional problems are worth mentioning. First, most DSGE models are nonlinear and linear
representations result from log-linear approximations. In the present paper we do not address
this issue; rather, we assume an exact linear representation for the economic variables. Sec-
ond, VAR estimation might entail a large truncation bias, as stressed in Chari, Kehoe and
McGrattan (2008). Although lag truncation is not the main focus here, we discuss a natural
extension of our deficiency measure to the case of the $K$-order VAR, $\delta^K$. Finite-order VAR
deficiency can be useful in practice, in that it provides a lower bound for the total bias due to
both informational deficiency and lag truncation.

In an application, we test a theory of news shocks. The model we employ is a New-
Keynesian DSGE, similar to the one used by Blanchard, Lorenzoni and L’Huillier (2013). It
features several frictions, such as internal habit formation in consumption, adjustment costs in
investment, variable capital utilization, Calvo price and wage stickiness. The model includes
seven exogenous sources of fluctuations: a news permanent shock and a surprise temporary
shock in technology, an investment-specific shock, a monetary policy shock, a shock to price
markups, a shock to wage markups and a shock to government expenditures. To select a
suitable VAR specification, we consider 10 candidates, with a number of variables varying
from two to seven. None of them is fundamental according to the theoretical model. Despite
this, four specifications pass the $\delta$-criterion for the news shock, $\delta^K$ being smaller than 0.05
for $K \geq 2$. We choose both a parsimonious four-variable specification, including TFP, GDP,
hours worked and the interest rate, and a seven-variable specification, including TFP, GDP, consumption, investment, hours, interest rate and inflation. We estimate the corresponding impulse response functions using US data for the sample 1954Q3-2015Q2. To identify the news shock, consistently with the model, we follow Beaudry and Portier (2015), i.e. we impose that (i) the surprise shock is the only shock affecting TFP on impact and (ii) the surprise and the news shock are the only ones affecting total factor productivity at a given horizon. Results show that, according to both VAR specifications, the theory performs reasonably well, even if it underrates the effects of news technology shocks on TFP.

The remainder of the paper is organized as follows. Section 2 introduces the theoretical framework and the measure, shows some examples and discusses the possible applications. Section 3 is devoted to the empirical application. Technical results are shown in Section 4. Section 5 concludes. The Appendix reports the details of the DSGE model used in Section 3.

2 Theory: Main ideas and two examples

2.1 Informational deficiency and the deficiency measure

We assume that the macroeconomic variables in the model have an exact Moving Average (MA) representation, possibly derived from a state-space representation, where the structural shocks propagate through linear impulse response functions. As a consequence, our results hold true for any theoretical model (not necessarily DSGE) which can be cast in MA form.\footnote{As observed above, a relevant problem for the validation of DSGE models through VARs is that most DSGE models are non-linear. The theoretical impulse response functions result from a linear approximation which in principle may be inaccurate. Since our focus is on informational deficiency, we do not address this issue here.}

Let us focus on the section of the macroeconomic model corresponding to the variables used in the VAR, i.e. the entries of the $n$-dimensional vector $x_t$. We assume that the vector $x_t$, possibly after transformations inducing stationarity, has the MA representation

$$x_t = \sum_{k=0}^{\infty} A_k u_{t-k} = A(L)u_t,$$

where $u_t = (u_{1,t} \cdots u_{q,t})'$ is a $q$-dimensional white noise vector of mutually orthogonal macroeconomic shocks, and $A(L) = \sum_{k=0}^{\infty} A_k L^k$ is an $n \times q$ matrix of square-summable impulse response functions.

Representation (1) is “structural” in the sense that the vector $u_t$ includes all of the exogenous shocks driving $x_t$. However, we do not assume that all of the shocks in $u_t$ have a structural economic interpretation: some of them may be statistical residuals, devoid of economic interest, arising from measurement errors. This enables us to evaluate VAR deficiency.
– and therefore its performance – with respect to the shocks of interest when actual variables are affected by measurement errors.

We do not assume that the number of variables $n$ is equal to the number of shocks $q$. In other words, representation (1) is not necessarily square. In particular, it can be “short”, with more shocks than variables, $q > n$. Short systems are relevant for applied work for two reasons. First, several empirical analyses are based on small-scale VARs, with just two or three variables. If the economy is driven by a larger number of shocks, the above MA system will be short. Second, most variables are in practice affected by measurement errors and/or small shocks of limited economic interest, so that, even if we have as many variables as major structural shocks (or even more variables than shocks) the system may be short because measurement errors are included in the vector $u_t$.\(^3\)

Given model (1), we want to evaluate whether a VAR in $x_t$ conveys the information needed to recover the shocks of interest and the corresponding impulse response functions. In practice, the impulse response function obtained with a VAR are affected by estimation errors arising from the finiteness of the sample size. Such finiteness requires specification of low-order VARs, which might be affected by truncation bias. Since our main focus here is on the non-fundamentalness bias, we replace finite-sample, finite-order VARs with orthogonal projections on infinite-dimensional information spaces.

Within this conceptual framework, the first step of our procedure is to compute the orthogonal decomposition

$$x_t = P(x_t | H^x_{t-1}) + \epsilon_t,$$

(2)

where $H^x_{t}$ is the closed linear space $\text{span}(x_{1,t-k}, \ldots, x_{n,t-k}, k = 0, \ldots, \infty)$ and $\epsilon_t$ is the Wold innovation.

The second step is to project $u_{it}$ onto the entries of $\epsilon_t$:

$$u_{it} = M\epsilon_t + e_{it}.$$

(3)

The variance of the above residual measures the approximation error. Informational deficiency is defined as the fraction of unexplained variance in the above projection:\(^4\)

$$\delta_i = \sigma^2_{\epsilon_i}/\sigma^2_{u_i}.$$  

(4)

The deficiency measure $\delta_i$ can be computed from the theoretical model (1), that is from $A(L)$, according to the formula provided in Section 4.

\(^{3}\)“Tall” systems, i.e. systems with more variables than shocks, are also interesting from a theoretical point of view, but are unlikely to occur in practice, because of measurement errors. We shall not consider them further in the present work.

\(^{4}\)For simplicity of notation we do not make explicit the dependence of $\delta_i$ on $x_t$.  

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We say that $x_t$ is informationally sufficient for $u_{it}$ if and only if $u_{it}$ is a linear combination of the entries of $\epsilon_t$, i.e. $\delta_i = 0$. In Section 4 we show that projecting $u_{it}$ onto the entries of $\epsilon_t$ is equivalent to projecting it onto the VAR information set $H^x_t$. It follows that we have sufficiency for $u_{it}$ if and only if $u_{it} \in H^x_t$.

By the very definition of informational sufficiency, if it is possible to obtain the vector $M$, then an informationally sufficient VAR for $u_{it}$ delivers $u_{it}$ without error, whereas an informationally deficient VAR for $u_{it}$ produces an approximation, whose error is measured by $\delta_i$.

However, the ultimate goal of the validation procedure are the impulse response functions, rather than the shock $u_{it}$ itself. In Section 4 we show that a VAR, which is sufficient for $u_{it}$ (but possibly deficient for the other structural shocks) and correctly identifies $u_{it}$ delivers the correct impulse response functions. Hence $\delta_i$ provides a meaningful indication about the performance of the theoretical VAR in approximating the impulse response functions. Of course, in practical situations we do not have theoretical VARs, but only finite-sample, finite-order VARs, which are affected by estimation and lag-truncation errors. Such real world VARs provide estimates whose asymptotic bias is measured by $\delta_i$, as the sample size and the truncation lag increase at appropriate rates.

Although the truncation bias is not our focus here, the deficiency measure can be naturally extended to the case of finite-order VARs. Deficiency of a VAR($K$) with respect to $u_{it}$, denoted by $\delta^K_i$, is given by the fraction of unexplained variance of the projection of $u_{it}$ onto the truncated VAR information space spanned by present and past values of the $x$’s, until the maximum lag $K$. By its very definition, the sequence $\delta^K_i$ is nonincreasing in $K$. The finite-order deficiency $\delta^K_i$ measures the total asymptotic bias due to deficiency plus lag truncation, as far as estimation of $u_{it}$ is concerned. Unfortunately, this is not true for the impulse response functions, for which the total bias may be larger (see Section 4 for details). Despite this, $\delta^K_i$ can prove useful in the context of a validation exercise, in that, being non-increasing in $K$, it represents a lower bound for the total bias.

### 2.2 Beyond the Poor Man’s Condition

How does our deficiency measure relate to fundamentalness and existing fundamentalness conditions? We have fundamentalness when all of the shocks in $u_t$ belong to the econometrician information set, i.e. $u_{it} \in H^x_t$, for all $i$. Hence we have fundamentalness if, and only if, the VAR is informationally sufficient for all shocks, that is $\delta_i = 0$ for all $i$. Sufficient information is then a notion of “partial fundamentalness”, a straightforward shock-specific generalization of the fundamentalness concept.

Note that short systems are never fundamental (see Section 4, Proposition 1). This is quite
intuitive: if we have just \( n \) variables we cannot estimate consistently more than \( n \) orthogonal shocks. Square systems, as is well known, can be either fundamental or not, depending on the roots of the determinant of \( A(L) \): we have fundamentalness if there are no roots smaller than 1 in modulus.

Fernandez-Villaverde et al. (2007) propose a fundamentalness condition, the PMC, based on the state-space representation of the economy. Consider the following linear equilibrium representation of a DSGE model

\[
\begin{align*}
    s_t &= As_{t-1} + Bu_t \\
    x_t &= Cs_{t-1} + Du_t
\end{align*}
\]  

(5)  

(6)

where \( s_t \) is an \( m \)-dimensional vector of stationary “state” variables, \( x_t \) is the \( n \)-dimensional vector of variables observed by the econometrician, and \( u_t \) is the \( q \)-dimensional vector of shocks with \( q \leq m \). \( A, B, C \) and \( D \) are conformable matrices of parameters, \( B \) has a left inverse \( B^{-1} \) such that \( B^{-1}B = I_q \). Representation (5)-(6) can always be cast in form (1). If the matrix \( D \) is square (this implies that the system is square, \( q = n \)) and invertible, the matrix \( A(L) \) appearing in representation (1) can be written as

\[
A(L) = DB^{-1} \left[ I - (A - BD^{-1}C)L \right] (I - AL)^{-1}B.
\]  

(7)

The PMC is that all the eigenvalues of the matrix \( A - BD^{-1}C \) are strictly less than one in modulus. It is easily seen that, if the PMC holds, the MA representation of \( x_t \) is invertible and \( u_t \) can be represented as a linear combination of the present and past values of \( x_t \). In other words, the PMC implies fundamentalness, i.e. sufficient information for all shocks.\(^5\) Hence, if the system is square, and the PMC holds, then \( \delta_i = 0 \) for all \( i \).

Summing up, the deficiency measure can be regarded as a generalization of the PMC (as well as other existing fundamentalness conditions), both because it is shock specific and because it provides information about the “degree” of non-fundamentalness. This generalization is very important for applied work. As anticipated above, short systems are never fundamental (and the PMC is not even defined in this case). By contrast, we show below that we may have small deficiency, or even sufficiency, for a single shock of interest, when \( q > n \). This implies a relevant fact, which is not well known in the literature: small-scale VARs can in principle be successfully employed even when the number of shocks driving the economy is large.

As for square systems, let us stress that non-fundamental structural MA representations, far from being an oddity, are common in macroeconomic models. They may arise from slow diffusion of technical change (Lippi and Reichlin, 1994b), news shocks (Sims, 2012, Forni, Gambetti and Sala, 2014, Beaudry and Portier, 2015), fiscal foresight (Leeper, Walker and

\(^5\)About the converse implication see Franchi and Paruolo (2015).
Young, 2013), noise shocks (Forni, Lippi, Gambetti and Sala, 2013a, 2013b). The seven-shocks DSGE model of Section 3, for instance, produces a non-fundamental representation for our seven-variable VAR specification. As a consequence, the PMC does not hold. Despite this, the VAR is almost sufficient for several shocks, including the news shock.

Sims (2012) makes the point that a VAR may perform reasonably well even if fundamentalness does not hold. With his words, non-fundamentalness “should not be thought of as an “either/or” proposition – even if the model has a non-invertibility, the wedge between VAR innovations and economic shocks may be small, and structural VARs may nonetheless perform reliably” (Sims, 2012, abstract). Both Beaudry and Portier (2015) and the present work provide further evidence about this fact. Our deficiency measure can be regarded as a formalization of the notion of “wedge between VAR innovations and economic shocks” discussed in Sims’ paper.

2.3 Two examples

We consider two simple examples that illustrate the concept of sufficient information (partial fundamentalness) and approximate sufficient information.

Example 1: Partial fundamentalness in a square system

Let us assume that output deviates from its potential value because of a demand shock \( d_t \) inducing temporary fluctuations, and reacts negatively to the interest rate \( r_t \), expressed in mean deviation, with a one-period delay. Precisely, the output gap \( y_t \) is given by

\[
y_t = (1 + \alpha L)d_t - \beta r_{t-1},
\]

where \( \alpha \) and \( \beta \) are positive. The central bank aims at stabilizing output by responding to output gap deviations, so that the interest rate follows the rule

\[
r_t = \gamma y_t + v_t,
\]

where \( v_t \) is a discretionary monetary policy shock and \( \gamma > 0 \). The structural MA representation for the output gap and the interest rate is then

\[
\begin{pmatrix}
y_t \\
r_t
\end{pmatrix} = \frac{1}{1 + \gamma \beta L} \begin{pmatrix}
1 + \alpha L & -\beta L \\
\gamma(1 + \alpha L) & 1
\end{pmatrix} \begin{pmatrix}
d_t \\
v_t
\end{pmatrix}.
\]

Here the determinant of the MA matrix is \((1 + \alpha L)\), which vanishes for \( L = -1/\alpha \), so that the representation is non-fundamental if \( |\alpha| > 1 \). From the policy rule we see that \( v_t = r_t - \gamma y_t \), so that the monetary policy shock can be recovered from the present values of the variables included in the VAR, irrespective of \( \alpha \) (of course, \( d_t \) cannot be found from the \( x \)'s if \( |\alpha| > 1 \)).
What happens when the above model is non-fundamental ($\alpha > 1$) and the econometrician tries to estimate the monetary policy shock and the related impulse-response functions? To answer this question we generated 1000 artificial data sets with 200 time observations from (8), with $\alpha = 3$, $\gamma = 0.4$, $\beta = 1$ and standard normal shocks. We then estimated for each data set a VAR with 4 lags and identified by imposing a standard Cholesky, lower triangular impact effect matrix, consistently with the model.

Figure 1 displays the true impulse response functions (red solid lines) along with the median (black dashed lines), the 5-th and the 95-th percentiles (grey area) of the distribution of the estimated impulse-response function to the demand shock $d_t$ (first column) and the monetary policy shock $v_t$ (second column). In the lower panels we report the distributions of the correlation coefficients between the estimated shocks and the true shocks.

The figure shows clearly that the impulse response functions are very poorly estimated for $d_t$, but very precisely for $v_t$. A similar result holds for the shocks themselves: the distribution of the correlation coefficients is very close to 1 for $v_t$ and far from 1 for $d_t$.

Let us now have a look at the true and estimated variance decomposition. Table 1 shows the fraction of the forecast error variance of $y_t$ and $r_t$ accounted for by the monetary policy shock. The contribution of the monetary policy shock to total variance is severely underestimated on impact, slightly underestimated at horizon 1 and well estimated at longer horizons. We shall come back on forecast error variance estimation in Section 4.

Table 2 shows the values of $\delta^K_d$, $\delta^K_v$, $K = 1, 4, 1000$. The VAR is dramatically deficient for the demand shock, consistently with Figure 1, but exhibits perfect information for the second shock, the monetary policy shock. Note that $x_t$ must be deficient for the demand shock, since the MA representation is non-fundamental.

**Example 2: Partial approximate fundamentalness in a short system**

Partial approximate sufficiency, far from being a statistical curiosity, is relevant in practice. As already noticed, most observed variables are likely affected by small macroeconomic shocks and/or measurement errors. Owing to these minor shocks, the applied researcher is usually faced with short systems, which are necessarily non-fundamental, but may be approximately sufficient for the shocks of interest.

As an example, consider the following news shock model, similar to the one used in Forni, Gambetti and Sala (2014). Total factor productivity, $a_t$, follows the slow diffusion process

$$a_t = a_{t-1} + \alpha \varepsilon_t + \varepsilon_{t-1}$$  \hfill (9)

where $0 \leq \alpha < 1$. 


The representative consumer maximizes
\[
E_t \sum_{t=0}^{\infty} \beta^t c_t,
\]
where \(E_t\) denotes expectation at time \(t\), \(c_t\) is consumption and \(\beta\) is a discount factor, subject to the constraint \(c_t + \bar{p}_t n_{t+1} = (\bar{p}_t + a_t)n_t\), where \(\bar{p}_t\) is the price of a share, \(n_t\) is the number of shares and \((\bar{p}_t + a_t)n_t\) is the total amount of resources available at time \(t\). The equilibrium value for asset prices is given by:
\[
\bar{p}_t = \sum_{j=1}^{\infty} \beta^j E_t a_t + j.
\]

Using (9), we see that \(E_t a_{t+k} = a_t + \varepsilon_t\) for all \(k > 0\). Hence, \(\bar{p}_t = (a_t + \varepsilon_t)\beta/(1 - \beta)\) and \(\Delta \bar{p}_t = b(1 + \alpha)\varepsilon_t\), where \(b = \beta/(1 - \beta)\). Let us assume further that actual prices \(p_t\) are subject to a temporary deviation from the equilibrium, driven by the shock \(d_t\), so that \(p_t = \bar{p}_t + \gamma d_t\).

In addition, let us add an orthogonal measurement error \(e_t\) to the technology variable \(a^*\), observed by the econometrician. The structural MA representation of \(\Delta a^*_t\) and \(\Delta p_t\) is short, because we have three shocks and just two variables:
\[
\begin{pmatrix}
\Delta a^*_t \\
\Delta p_t
\end{pmatrix} =
\begin{pmatrix}
\alpha + L & 0 & \theta(1 - L) \\
b(1 + \alpha) & \gamma(1 - L) & 0
\end{pmatrix}
\begin{pmatrix}
\varepsilon_t \\
d_t \\
e_t
\end{pmatrix}.
\]

We assume unit variance shocks, so we add a scaling factor \(\theta\) to the impulse response function of \(e_t\) to control for the size of the measurement error. We set \(\beta = 0.99\), \(\alpha = 0.5\) and \(\gamma = 20\). Moreover, we set \(\theta = 0.5\), so that the measurement error is large (it explains more than 25% of the total variance of \(\Delta a^*_t\)).

Figure 2 shows the estimation results obtained by a Monte Carlo exercise with \(T = 200\), i.i.d. unit variance Gaussian shocks and 1000 artificial data sets. The VAR is estimated with 4 lags and is identified by assuming that \(\varepsilon_t\) is the only one shock affecting \(a^*_t\) in the long run, consistently with the model. The estimates of the technology shock \(\varepsilon_t\) and the related impulse response functions are fairly good, even if a small distortion is visible. On the contrary, the temporary shock to stock prices \(d_t\) and the associated responses are poorly estimated. Correspondingly, the deficiency measure is 0.03 for \(\varepsilon_t\) and 0.97 for \(d_t\) (see Table 3).

### 2.4 DSGE models validation

To get the intuition of the usefulness of the measure, consider a situation where a researcher has two models with different predictions in terms of the response of a specific variable of interest.
As an example, consider the technology-hours debate. The RBC model predicts that hours should increase while the New-Keynesian model predicts that hours should fall in response to a contemporaneous and permanent technology shock. Gali (1999) estimates a bivariate VAR in labor productivity and hours and identifies the technology shock as the only one shock driving labor productivity in the long run. He then checks whether hours respond positively, as implied by the RBC model, or negatively, as implied by sticky price models. In order for the results obtained from the VAR analysis to be meaningful in discriminating between models, it is important to check that the deficiency measure associated with a bivariate VAR in labor productivity and hours is close to zero in both models. Only in that case can the VAR evidence support one of the two theories. If on the other hand the measure is large for at least one theoretical model, the comparison based on VAR analysis is inappropriate.

Validation should therefore be performed through the following steps.

1. Consider a calibrated/estimated DSGE model and its linear equilibrium representation as in (5)-(6) or (1). Select the shock of interest \( u_{it} \).

2. Explore different VAR specifications (including variables represented in the model) by computing \( \delta^K_i \). If the measure is larger than a pre-specified threshold (e.g. 0.05 or 0.1), consider a different specification. Otherwise, go to the following step.

3. Evaluate the lag-truncation and the small-sample bias as follows. First, generate from the model artificial series with the same length as the sample to be used for validation. Next verify whether a VAR\((K)\) is able to reproduce the theoretical impulse functions. If the result is acceptable, go to the final step.

4. Verify whether the theoretical impulse response functions lie within the VAR confidence bands. If they do, the model is validated. If they do not, there is something wrong either in the values of the parameters or in the model itself.

As observed above, in this paper we abstract from estimation issues. However, in practice the true VAR population parameters are unknown and must be estimated with a \( T \)-dimensional sample. If \( T \) is small, large-scale VAR specifications may fail because of the estimation errors. .... We recommend using the most parsimonious specification among those having deficiency measure below the threshold level.
3 Empirics: Validating a theory of news shocks

3.1 The economic model

In this Section, we assess a theory of news shocks to technology. The model is a New-Keynesian DSGE, similar to the one used by Blanchard, Lorenzoni and L’Huillier (2013, BLL henceforth). We choose a news shock model for our application since such models are often regarded as incompatible with VARs because of non-fundamentalness problems (see e.g. Christiano, Motto and Rostagno, 2014).

The model features several frictions, such as internal habit formation in consumption, adjustment costs in investment, variable capital utilization, Calvo price and wage stickiness. The model also features seven exogenous sources of fluctuations, namely, a news shock and a surprise shock in technology, an investment-specific shock, a monetary policy shock, a shock to price markups, a shock to wage markups and a shock to government expenditures.

We assume that the logarithm of technology follows the process

\[ a_t = a_{t-1} + \varepsilon_{t-4} + (1 - L)T_t \quad (13) \]

\[ T_t = \rho T_{t-1} + \nu_t \quad (14) \]

where \( \varepsilon_t \) is a news shock, which is observed by agents at time \( t \), but will be reflected in \( a_t \) at time \( t + 4 \). The component \( T_t \) is a temporary component driven by the surprise technology shock \( \nu_t \). Our validation exercise focuses on the news shock \( \varepsilon_t \).

As for the parameters, some of them are calibrated using the posterior mean values estimated by BLL (see Table 4). The remaining ones are estimated using Bayesian techniques; estimation results are reported in Table 5. A complete description of the model and the estimation details are reported in Appendix A.

3.2 The data and the structural VAR

For the VAR validation exercise, we use US quarterly data on Total Factor Productivity, real per-capita GDP, real per-capita consumption of non-durables and services, real per-capita investment, per-capita hours worked, the federal funds rate and the inflation rate. The sample span is 1954Q3-2015Q2. Further details about the data and their treatment are provided in Appendix B.

All VAR estimates are Bayesian estimates with diffuse prior. Data are taken in levels. The number of lags is 4 unless otherwise stated. Point estimates of the impulse response functions are obtained as averages of the posterior distribution across 500 draws, unless otherwise stated.

Following Beaudry and Portier (2015), the news shock is identified by imposing that (i) no shocks other than the technology surprise shock affect TFP on impact; (ii) surprise and
news shocks are the only ones affecting TFP at a given horizon.\footnote{Here we use the five year horizon (lag 20). Other choices in the range 12-28 lags produce very similar results. Longer horizons, such as the ten-year horizon, produce a larger estimation bias in the present case.} Restrictions (i) and (ii) are just identifying and, of course, are consistent with the theory (see the above equation). This identification scheme is the same as the one used in Forni, Gambetti and Sala (2014), where condition (ii) is replaced by the equivalent condition that the effect of news on TFP at the given horizon is maximized.

### 3.3 Assessing deficiency of alternative VAR specifications

To select the set of variables to use in the validation exercise we examine ten candidate specifications:

- **S1**: TFP, consumption.
- **S2**: TFP, investment.
- **S3**: TFP, consumption, hours worked, interest rate.
- **S4**: TFP, GDP, consumption, hours, interest rate.
- **S5**: TFP, GDP, consumption, hours, inflation.
- **S6**: TFP, GDP, investment, hours, interest rate.
- **S7**: TFP, GDP, investment, hours, inflation.
- **S8**: TFP, GDP, investment, hours, interest rate and inflation.
- **S9**: TFP, GDP, consumption, investment, hours, interest rate.
- **S10**: TFP, GDP, consumption, investment, hours, interest rate and inflation.

The informational deficiency measure is computed on the variables transformed to obtain stationarity (see Appendix A). The structural shocks cannot be fundamental for specifications 1-9, since they are short. Specification S10 is square, so that in principle we might have fundamentalness. For each specification, we compute the value of $\delta^K$, $K = 1, 2, 4, 12, 1000$. The value of $\delta(1000)$ is taken as our approximation of $\delta$. Even if we are interested in the news shock only, we compute $\delta$ for all shocks for illustrative purposes.

Table 6 shows the results. A few observations are in order. First, the structural shocks are non-fundamental for the variables in S10, since for some $i$ we have $\delta_i > 0$. Of course the PMC is not satisfied, the largest eigenvalue being 1.13. Hence each one of the specification...
considered have a non-fundamental representation in the structural shocks. Second, despite non-fundamentalness, several specifications exhibit very low deficiency for a few specific shocks. For instance, in the 4-variable specification S3, \( \delta_i < 0.01 \) for both the news and the surprise technology shocks (columns 1 and 2, respectively). Third, a specification may be highly deficient for some shocks and sufficient for other shocks. For instance specification S9, in which inflation is absent, is sufficient, or almost sufficient, for four shocks out of seven, but (not surprisingly) has a value of \( \delta \) as high as 0.78 for the third shock, which is the price markup shock.

Focusing on our shock of interest (first column), we see that there are four specifications, namely S3, S4, S9 and S10, exhibiting a value of \( \delta \) smaller than 0.05. Table 7 shows the values of \( \delta^K \) for the news shocks, \( K = 1, 2, 4, 12, 1000 \), for these four specifications. As already observed, these numbers should be interpreted as lower bounds for the total bias due to non-fundamentalness and lag truncation and therefore can provide some guidance for the number of lags to use in the VAR. We see from the Table that including just one lag is inappropriate for all specifications. By contrast, for \( K \geq 2 \) the value of \( \delta \) is smaller than 0.05. The most interesting specifications are S3, which is parsimonious, and S10, which enables us to consider more variables for the validation exercise. We use both of them in the following analysis.

Before considering S3 and S10 in more detail, let us see what happens if, ignoring deficiency, we use instead S1 or S2 for validation purposes. Figure 3 shows the VAR results for the news shock obtained with real data. The top panels refer to S1 while the bottom panels refer to S2. The black solid lines are the empirical impulse response functions to unit-variance shocks. The blue dashed lines are the theoretical impulse response functions to unit-variance shocks. According to S1, the theoretical model should be rejected, because the effects of news shocks are largely overstated. According to S2, the theoretical model should again be rejected, but for the opposite reason: the effect of news is understated.

### 3.4 Evaluating the truncation bias and the estimation bias

Now let us focus on our preferred specifications S3 and S10. Chari et al. (2008) highlights that the VAR may be affected by large truncation and estimation bias. To evaluate the different sources of bias involved in VAR estimation, we generate artificial data for the variables in levels from the model and estimate the VAR on artificial data in order to see whether the VAR is able to reproduce the true impulse response functions. We use 500 Monte Carlo replications. For each artificial data set we estimate a Bayesian VAR with diffuse priors and take the average of the posterior impulse responses over 50 draws from the posterior distribution of the VAR parameters.

Figures 4 report the results for the four-variable specification S3. The blue dashed lines are
the theoretical impulse response functions. The red thick solid lines are obtained with 5000 time observations and 12 lags. We take them as showing the asymptotic deficiency bias. The green dotted lines are obtained with 5000 time observations and 4 lags. We interpret them as showing the total asymptotic bias due to deficiency and lag truncation. The black thin solid lines are obtained with 243 observations and 4 lags. They show the total bias due to deficiency, truncation and small sample estimation. The dark gray and light gray areas are the 68% and 90% posterior probability intervals obtained with 4 lags and 243 observations.

We see from the figure that (a) the deficiency bias is negligible, in that the red lines are almost identical to the blue dashed lines; (b) the truncation bias is fairly small, even if it is clearly visible in the TFP and the hours-worked panels (green-dotted versus red-solid lines); (c) the small sample bias is sizable (black-solid versus green-dotted lines). The total bias is relevant, particularly for consumption and hours worked. However, the theoretical impulse response functions lie within the narrower bands, except for the corner of the TFP response at lag 4.

Figure 5 reports results for the seven-variable specification. The conclusions are similar to the previous ones, even if in this case the total bias is somewhat less pronounced at medium-long horizons. We conclude that both VAR specifications can be used for validation purposes.

3.5 Validating the theory

We now come to the validation exercise. We estimate the VARs S3 and S10 with real data by using the method and the identification restrictions described above. Figure 6 plots the results for the four-variable specification. As before, the solid black line is the average of the posterior distribution, the dark and light gray areas represent the 68% and 90% probability intervals, respectively and the blue dashed line is the impulse response function of the economic model. For both specifications, the theoretical impulse response functions have the correct signs and lie within the 90% bands for all variables. However, the sudden reaction of TFP at lag 4 is clearly at odds with the VAR estimates, where the empirical impulse response function increases gradually, according to a typical S-shape. Moreover, its long-run effect is much larger than the theoretical one. This problem might be fixed by specifying a more flexible exogenous process for technology. The reaction of hours worked is anticipated with respect to the empirical one, whereas the converse is true for the interest rate.

Figure 7 plots the results for the seven-variable specification. The above results are confirmed. The long-run reaction of TFP is now very close to the lower bound of the 68% posterior

\footnote{This is in order to replicate the sample size in US data.}
probability interval. The reaction of GDP is almost identical to the empirical one. Notice however that, were the long-run effect on TFP larger, the effect on GDP and consumption would likely be overstated by the model.

Let us now focus on investment and inflation, two variables not present in specification S3. The signs of the impulse response functions are correct. However, the reaction of investment is understated by the model in the short run. Moreover, the effect on inflation predicted by the model at horizon 30 is zero, whereas the empirical one is positive.

Our overall evaluation is that the model performs reasonably well but clearly understates the long-run effects of technology news on TFP.

4 Some structural VAR theory for non-fundamental and “short” models

4.1 Fundamentalness: definition and standard results

Let us begin by reviewing the definition of fundamentalness and a few related results.

**Definition 1** (Fundamentalness). We say that \( u_t \) is fundamental for \( x_t \) if and only if \( u_t \in H_t^x \), where

\[
H_t^x = \text{span}(x_{1,t-k}, \ldots, x_{n,t-k}, k = 0, \ldots, \infty)\]

Now, consider the theoretical projection equation of \( x_t \) on its past history, i.e. equation (2). The Wold representation of \( x_t \) is

\[
x_t = B(L)\epsilon_t, \tag{15}
\]

where \( B(0) = I_n \).

The following result is standard in time series theory.

**Proposition 1.** \( u_t \) is fundamental for \( x_t \) if and only if there exist a nonsingular matrix \( Q \) such that \( u_t = Q\epsilon_t \).

It is apparent from the above condition that fundamentalness cannot hold if the system is short. In this case, a matrix \( Q \) satisfying Proposition 1 does not exist, since for any \( q \times n \) matrix \( Q \), with \( q > n \), the entries of \( Q\epsilon_t \) are linearly dependent, whereas the entries of \( u_t \) are mutually orthogonal.

By contrast, in the square case \( n = q \) fundamentalness clearly holds if the impulse response function matrix \( A(L) \) is invertible; for, in this case, we can write

\[
A(L)^{-1}x_t = u_t,
\]

so that the condition defining fundamentalness is satisfied. It is easily seen from (15) that in this case Proposition 1 holds with \( Q = A(0)^{-1} \) and \( A(L) = B(L)A(0) \).
In the particular case of $A(L)$ being a matrix of rational functions, fundamentalness of $u_t$ for $x_t$ is equivalent to the following condition (see e.g. Rozanov, 1967, Ch. 2).

**Condition R.** The rank of $A(z)$ is $q$ for all complex numbers $z$ such that $|z| < 1$.

When $A(L)$ is a square matrix the above condition reduces to the well known condition that the determinant of $A(z)$ has no roots smaller than one in modulus. Fundamentalness is therefore slightly different from invertibility, since invertibility rules out also roots with modulus equal to 1. Hence invertibility implies fundamentalness, whereas the converse is not true.\(^8\)

### 4.2 VAR deficiency and sufficient information

For simplicity we shall assume here that the target of VAR estimation is the single shock of interest $u_{it}$ (along with the corresponding impulse response functions). The generalization to any subvector $v_t$ of $u_t$, including $s \leq q$ shocks, is straightforward.

Let us go back to the VAR representation of $x_t$, i.e. equation (2), and the projection equation (3). The following proposition says that the structural VAR strategy, i.e. approximating $u_{it}$ by means of the VAR residuals, is optimal in the sense that it provides the best linear approximation, given the VAR information set.

**Proposition 2** (Optimality of the structural VAR procedure). The projection of $u_{it}$ onto the entries of $\epsilon_t$, i.e. $M\epsilon_t$, is equal to the projection of $u_{it}$ onto $H^x_t$.

Proof. From (3) it is seen that $H^x_t$ is the direct sum of the two orthogonal spaces $H^x_{t-1}$ and $\text{span}(\epsilon_{jt}, j = 1, \ldots, n)$. Hence $P(u_{it}|H^x_t) = P(u_{it}|\epsilon_{jt}, j = 1, \ldots, n) + P(u_{it}|H^x_{t-1})$. Since $u_{it}$ is orthogonal to the past values of the $x$'s, the latter projection is zero. Hence $P(u_{it}|H^x_t) = P(u_{it}|\epsilon_{jt}, j = 1, \ldots, n) = M\epsilon_t$. QED

Proposition 2 motivates the following definitions.

**Definition 2** (VAR deficiency and sufficient information). The informational deficiency of $x_t$ (and the related VAR information set $H^x_t$) with respect to $u_{it}$ is

$$\delta_i = \text{var}[u_{it} - P(u_{it}|H^x_t)]/\sigma^2_{u_{it}} = \sigma^2_{\epsilon_i}/\sigma^2_{u_{it}}.$$  

We say that $x_t$ is informationally sufficient for $u_{it}$ if and only if $\delta_i = 0$, i.e. $u_{it} \in H^x_t$, or, equivalently, $u_{it} = M\epsilon_t$.

As an immediate consequence of Definitions 1 and 2, we have the following result.

---

\(^8\)The unit root case is economically interesting in that, if $x_t = \Delta X_t$ and the determinant of $A(z)$ vanishes for $z = 1$, then the entries of $X_t$ are cointegrated. Non-invertibility implies that $x_t$ does not have a VAR representation and VAR estimates do not have good properties. However this problem can be solved by estimating an ECM or a VAR in the levels $X_t$.  

---
Proposition 3. \( u_t \) is fundamental for \( x_t \) if and only if \( x_t \) is informationally sufficient for \( u_{it} \), \( i = 1, \ldots, q \), or equivalently, \( \delta_i = 0 \) for all \( i \).

4.3 Partial sufficiency: IRFs

Until now we have focused on the conditions under which the VAR is able to recover the shock \( u_{it} \). However, the ultimate goal of the VAR validation procedure are the impulse response functions, rather than the shock itself. Hence a basic question in our framework is the following.

Let the VAR be sufficient for \( u_{it} \) and the identification restrictions be correct. Are the impulse response functions obtained from the VAR equal to the theoretical ones? An additional, related question is: is the forecast error variance decomposition equal to its theoretical counterpart? Standard results in VAR identification theory guarantee a positive answer to both questions when the MA representation is (globally) fundamental. But what happens if this is not the case?

The structural VAR procedure consists in inverting the VAR representation to estimate the Wold representation (15) and choosing identification restrictions which deliver an “identification matrix”, say \( Q \). The structural shocks are then obtained as \( v_t = Q \epsilon_t \) and the corresponding impulse response functions as \( A\ast(L) = B(L)Q^{-1} \).

Let us assume that \( H^\ast \) is sufficient for \( u_{it} \), so that \( u_{it} \) can be recovered as the linear combination \( M\epsilon_t \). We shall use the following definition.

Definition 4 (Correct identification). An identification matrix is a nonsingular \( n \times n \) matrix \( Q \) such that \( Q\Sigma\epsilon Q' \) is diagonal, i.e. the entries of \( v_t = Q\epsilon_t \) are orthogonal. An identification matrix is correct for \( u_{it} \) if and only if \( v_t = Q\epsilon_t \) is such that \( v_{ht} = u_{it} \) for some \( 1 \leq h \leq n \), i.e., denoting with \( Q_h \) the \( h \)-th line of \( Q \), \( Q_h = M \).

If \( Q \) is a correct identification matrix, we can write the impulse response function representation derived from the VAR as

\[
x_t = A\ast(L)v_t = A\ast_h(L)v_{ht} + A\ast_{-h}(L)z_t = A\ast_h(L)u_{it} + A\ast_{-h}(L)z_t,
\]

where \( A\ast_h(L) \) is the \( h \)-th column of \( A\ast(L) \), \( A\ast_{-h}(L) \) is the \( n \times n-1 \) matrix obtained by eliminating the \( h \)-th column from \( A\ast(L) \) and \( z_t = (v_{1t} \cdots v_{h-1,t} v_{h+1,t} \cdots v_{nt})' \). \( A\ast_h(L) \) is the vector of impulse response functions derived from the VAR – let us say the “empirical” impulse response functions, even if, of course, we are speaking of the population VAR (with infinite sample size).

Now let \( A_i(L) \) be the \( i \)-th column of \( A(L) \) and \( A_{-i}(L) \) be the \( n \times q-1 \) matrix obtained by eliminating the \( i \)-th column from \( A(L) \). The structural MA representation can then be written as

\[
x_t = A(L)u_t = A_i(L)u_{it} + A_{-i}(L)w_t,
\]
where \( w_t = (u_{1t} \cdots u_{i-1,t} u_{i+1,t} \cdots u_{qt})' \). \( A_i(L) \) is the vector of the “true” impulse response functions.

**Proposition 4.** Let \( x_t \) and the related VAR be informationally sufficient, and the identification matrix be correct for \( u_{it} \). Then the empirical impulse response functions are equal to the true impulse response functions, i.e. \( A_i^*(L) = A_i(L) \) for some \( h, 1 \leq h \leq n \).

Proof. Let us first observe that the entries of \( v_t \) are orthogonal at all leads and lags, since \( v_t = Q \epsilon_t \) is a vector white noise and \( Q \) is an identification matrix. It follows that \( u_{it} \) is orthogonal to the entries of \( z_t \) at all leads and lags. Moreover, by the assumptions of model (1), \( u_{it} \) is also orthogonal to \( u_{jt}, j \neq i \), and therefore to the entries of \( w_t \), at all leads and lags. From (16) and (17) we get \( A_i^*(L)u_{it} + A_{-i}^*(L)z_t = A_i(L)u_{it} + A_{-i}(L)w_t \). Projecting both sides onto \( u_{i,t-k}, k \geq 0 \) we get \( A_i^*(L)u_{it} = A_i(L)u_{it} \), which implies the result. QED

The equality result in Proposition 4 translates into a consistency result for real-world, finite-sample VARs, provided that the parameters of the population VAR are estimated consistently, and the truncation lag increases with the sample size, following a consistent information criterion.

### 4.4 Partial sufficiency: variance decomposition

Partial identification has also implications in term of variance decomposition. It is well known that \( H^e \cap H^u \) and, if \( u_t \) is non-fundamental for \( x_t \), \( H^e_t \subset H^u_t \). As a consequence, in the non-fundamental case, the prediction error of \( v_t \) is larger than that of \( u_t \). Precisely, for any horizon \( s \geq 0 \), we have

\[
\text{var} [P(x_{i,t+s} | H_{t-1}^u) - x_{i,t+s}] \leq \text{var} [P(x_{i,t+s} | H_{t-1}^u) - x_{i,t+s}],
\]

and the inequality is strict at least for \( s = 0 \) if \( u_t \) is non-fundamental for \( x_t \). Hence if \( x_t \) is informationally sufficient for \( u_{it} \), but not for all shocks, then total forecast error variance is overestimated by the VAR model at short horizons. On the other hand, Proposition 4 implies that the impulse response functions of \( u_{it} \), and therefore the variance of the forecast errors, is estimated consistently. Putting things together, the fraction of total variance accounted for by \( u_{it} \), derived from the VAR, is downward biased, since the numerator is unbiased, whereas the denominator is upward biased.\(^9\)

An alternative variance decomposition, which is not affected by this bias, is obtained by using integrals of the spectral densities over suitable frequency bands (see e.g. Forni, Gambetti and Sala, 2016). Let \( A_{i,j}(L) \) and \( A_{i,j}^*(L) \), be the \( j \)-th elements of the matrices \( A_i(L), A_i^*(L) \),

\(^9\)If \( x_t = \Delta X_{it} \) a similar result holds for the decomposition of the forecast error variance of the level \( X_{i,t+s} \).

This explains the large estimation error, at horizon 0, reported in Table 1 for the variable \( r_t \).
respectively. As is well known, the variance of the component of \( x_{jt} \) which is attributable to \( v_{ht} \) can be computed as \( \sigma_{v_{ht}}^2 \int_0^{\pi} A_{-i,j}^{-}(e^{-i\theta})A_{-i,j}(e^{i\theta})d\theta/\pi \). If we are interested for instance in the variance of waves of business cycle periodicity, say between 8 and 32 quarters, the corresponding angular frequencies (with quarterly data) are \( \theta_1 = \pi/4 \) and \( \theta_2 = \pi/16 \) and the corresponding variance is \( \sigma_{v_{ht}}^2 \int_{\theta_1}^{\theta_2} A_{-i,j}^{-}(e^{-i\theta})A_{-i,j}(e^{i\theta})d\theta/\pi \). On the other hand, the total “cyclical” variance of \( x_{jt} \) is given by \( \int_{\theta_1}^{\theta_2} S_{x_j}(\theta)d\theta/\pi \), where \( S_{x_j}(\theta) \) denotes the spectral density of \( x_{jt} \). Hence the contribution of \( v_{ht} \) to the cyclical variance of \( x_{jt} \) is given by

\[
\frac{\sigma_{v_{ht}}^2 \int_{\theta_1}^{\theta_2} A_{h,j}^-(e^{-i\theta})A_{h,j}^+(e^{i\theta})d\theta}{\int_{\theta_1}^{\theta_2} S_{x_j}(\theta)d\theta}.
\]

Similarly, the contribution of \( u_{it} \) is given by

\[
\frac{\sigma_{u_{it}}^2 \int_{\theta_1}^{\theta_2} A_{i,j}(e^{-i\theta})A_{i,j}(e^{i\theta})d\theta}{\int_{\theta_1}^{\theta_2} S_{x_j}(\theta)d\theta}.
\]

Under the assumptions of Proposition 4, the numerators are equal, so that the ratios are equal. Therefore this kind of variance decomposition analysis is preferable to the standard one in that it is not biased in the case of partial fundamentalness.

### 4.5 Finite-order VAR deficiency

In this subsection we consider a quite natural extension of the deficiency measure to the case of finite-order VARs. Let us denote the VAR(\( K \)) information set as \( H_{x}^{K}(t) = \text{span}(x_{j,t-k}, j = 1, \ldots, n, k = 0, \ldots, K) \) and consider the orthogonal decompositions

\[
\begin{align*}
x_t & = P(x_t|H_{x}^{K}(t)) + \epsilon_{t}^{K} \\
u_{it} & = M^{K}\epsilon_{it}^{K} + e_{it}^{K}.
\end{align*}
\]

Proposition 2 still holds for the finite-order VAR.

**Proposition 2’** The projection of \( u_{it} \) onto the entries of \( \epsilon_{t}^{K} \), i.e. \( M^{K}\epsilon_{t}^{K} \), is equal to the projection of \( u_{it} \) onto \( H_{x}^{K}(t) \).

The proof is the same as that of Proposition 2, with \( \epsilon_{t}^{K} \) in place of \( \epsilon_{t} \) and \( H_{x}^{K}(t) \) in place of \( H_{x}^{\tau} \), \( \tau = t, t-1 \). The VAR(\( K \)) deficiency can then be defined as

\[
\delta_{t}^{K} = \text{var}[u_{it} - P(u_{it}|H_{x}^{K}(t))]/\sigma_{u_{it}}^2 = \sigma_{\epsilon_{t}^{K}}^2/\sigma_{u_{it}}^2.
\]

Correspondingly, we can say that \( H_{x}^{K}(t) \) is informationally sufficient for \( u_{it} \) if and only if \( \delta_{t}^{K} = 0 \), i.e. \( u_{it} \in H_{x}^{K}(t) \), or, equivalently, \( u_{it} = M^{K}\epsilon_{t}^{K} \). As \( K \) increases, the spaces \( H_{x}^{K}(t) \) are
nested, so that the sequence $\delta^K_i$ is non-increasing in $K$ and $\delta_i \leq \delta^K_i$ for any $K$. The difference $\delta^K_i - \delta_i$ provides a precise information about the effect of lag truncation on estimation of the shock $u_{it}$.

Unfortunately, this is not true for the impulse response functions, since Proposition 4 does not hold for the finite-order VAR. It might be the case that the $K$-order VAR is sufficient for $u_{it}$, but the corresponding impulse response functions are biased. Hence the difference $\delta^K_i - \delta_i$ cannot be regarded as a measure of the additional bias due to lag truncation. Nevertheless, $\delta^K_i$ deserves interest in validation exercises, in that it provides a lower bound for the overall bias due to non-fundamentalness and lag truncation.

4.6 Computing VAR deficiency

To compute VAR deficiency, a simple formula can be derived as follows. Let us write the projection equation of $u_{it}$ onto $H^x_t(K)$ as

$$u_{it} = P(u_{it}|H^x_t(K)) + e^K_{it} = Fy_t + e^K_{it},$$

where $y_t = (x'_{t-1} \cdots x_{t-K})'$ and $F = E(u_{it}y_t')\Sigma_y^{-1}$, $\Sigma_y$ being the variance covariance matrix of $y_t$. From (21) and the above equation we get

$$\delta^K_i = 1 - F \Sigma_y F'/\sigma^2_{u_i} = 1 - E(u_{it}y_t')\Sigma_y^{-1}E(u_{it}y'_t)/\sigma^2_{u_i}.$$

Using (17) it is easily seen that $E(u_{it}x'_t) = A_i(0)'$ and $E(u_{it}x'_{t-k}) = 0$ for all $k > 0$, so that $E(u_{it}y'_t) = (A_i(0)' 0 \cdots 0)$.

Hence

$$\delta^K_i = 1 - A_i(0)' GA_i(0)/\sigma^2_{u_i}, \quad (22)$$

This is because the VAR residuals in $\epsilon^K_t$ might be serially correlated. By inverting the finite-order VAR, we get the representation

$$x_t = B^K(L)e^K_t,$$

and, by imposing the identification constraints, we get the “shocks” $v^K_t = Qe^K_t$ and the corresponding impulse response functions $A^K(L) = B^K(L)Q^{-1}$. To get unbiasedness of these response functions we need the assumption of Proposition 4, along with the additional condition that $e^K_t$ is a vector white noise.

**Proposition 4’.** Let a VAR($K$) be informationally sufficient, and the identification matrix be correct for $u_{it}$. Assume further that the VAR residual $e^K_t$ is a vector white noise. Then the empirical impulse response functions are equal to the true impulse response functions, i.e. $A^K_i(L) = A_i(L)$.

We omit the proof, which is essentially the same as that of Proposition 4. Notice that, starting from the parameters of the economic model, we can check in principle whether serial correlation of $e^K_t$ is satisfied.
where $G$ is the $n \times n$ upper-left submatrix of $\Sigma_y^{-1}$, $\Sigma_y$ being

$$
\Sigma_y = \begin{pmatrix}
\Gamma_0 & \Gamma_1 & \cdots & \Gamma_K \\
\Gamma_{-1} & \Gamma_0 & \cdots & \Gamma_{K-1} \\
\vdots & \vdots & \ddots & \vdots \\
\Gamma_{-K} & \Gamma_{-K+1} & \cdots & \Gamma_0 
\end{pmatrix},
$$

where $\Gamma_k = E(x_t x_{t-k}')$, $k = 0, \ldots, K$. The covariance matrices of $x_t$ can easily be computed from the MA representation (1) by using the covariance generating function

$$
A(L)\Sigma_u A(L^{-1})' = \sum_{k=-\infty}^{\infty} \Gamma_k L^k.
$$

As for $\delta_i$, it can simply be approximated with any desired precision by using a suitably large $K$.

## 5 Conclusions

Structural VAR models can be used for the empirical validation of macroeconomic models even if the number of variables in the VAR is smaller than the number of structural shocks in the model, or, more generally, the impulse-response representation of the variables in the VAR is nonfundamental.

What is really necessary for the validation exercise to be meaningful is that the VAR conveys enough information to recover the shocks of interest and the related impulse response functions.

VAR deficiency for a given shock can be measured by $\delta_i$, i.e. the fraction of unexplained variance of the linear projection of this shock onto the VAR information set. Hence a crucial step of the validation procedure is to verify whether $\delta_i$ is acceptably small.

An exact formula for $\delta_i$ is

$$
\delta_i = 1 - \sigma_{u_i}^2 A_i(0)' \Sigma^{-1}_\epsilon A_i(0).
$$

This formula is obtained from (3) by observing that $M = E(u_{it}\epsilon_t')\Sigma^{-1}_\epsilon$ and noting that, by (2) and (1), $E(\epsilon_t u_{it}) = E(x_t u_{it}) = A_i(0)\sigma_{u_i}^2$. If the model can be written in the state-space form

$$
\begin{align*}
s_t &= A s_{t-1} + Bu_t \\
x_t &= H s_t
\end{align*}
$$

the matrix $\Sigma_\epsilon$ can be obtained from the Wold representation

$$
x_t = \{I_n + H(I_m - AL)^{-1}KL\} \epsilon_t
$$

where $K$ is the steady-state Kalman gain: $\Sigma_\epsilon$ is given by $HPH'$, where $P$ is the steady-state variance-covariance of the states.
For DSGE models including news or foresight shock, non-fundamentalness is endemic. Such models are often regarded as incompatible with VARs, in that a VAR representation in the structural shocks does not exist.

Hence we illustrate our ideas by conducting a validation exercise with a news shock DSGE model. We show that our VAR specification can be used for model validation, despite non-fundamentalness. We find that the DSGE model performs reasonably well in fitting the impulse response functions derived from US data.
References


Appendix A: The DSGE model

The model follows closely Blanchard, L’Huillier and Lorenzoni (2013). The preferences of the representative household are given by the utility function:

\[ E_t \left[ \sum_{t=0}^{\infty} \beta^t \left( \log (C_t - hC_{t-1}) - \frac{1}{1 + \varsigma} \int_0^1 N_{jt}^{1+\varsigma} dj \right) \right], \]

where \( C_t \) is consumption, the term \( hC_{t-1} \) captures internal habit formation, and \( N_{jt} \) is the supply of specialized labor of type \( j \). The household budget constraint is

\[ P_tC_t + P_tI_t + T_t + B_t + P_tC(U_t)\bar{K}_{t-1} = R_{t-1}B_{t-1} + Y_t + \int_0^1 W_{jt}N_{jt}dj + R^K_tK_t, \]

where \( P_t \) is the price level, \( T_t \) is a lump sum tax, \( B_t \) are holdings of one period bonds, \( R_t \) is the one period nominal interest rate, \( Y_t \) are aggregate profits, \( W_{jt} \) is the wage of specialized labor of type \( j \), \( N_{jt} \). \( R^K_t \) is the capital rental rate.

Households choose consumption, bond holdings, capital utilization, and investment each period so as to maximize their expected utility subject to the budget constraint and a standard no-Ponzi condition. Nominal bonds are in zero net supply, so market clearing in the bonds market requires \( B_t = 0 \).

The capital stock \( K_t \) is owned and rented by the representative household and the capital accumulation equation is

\[ \bar{K}_t = (1 - \delta)\bar{K}_{t-1} + D_t \left[ 1 - G(I_t/I_{t-1}) \right] I_t, \]

where \( \delta \) is the depreciation rate, \( D_t \) is a stochastic investment-specific technology parameter, and \( G \) is a quadratic adjustment cost in investment

\[ G(I_t/I_{t-1}) = \chi(I_t/I_{t-1} - \Gamma)^2/2, \]

where \( \Gamma \) is the long-run gross growth rate of TFP. The model features variable capacity utilization: the capital services supplied by the capital stock \( \bar{K}_{t-1} \) are \( K_t = U_t\bar{K}_{t-1} \), where \( U_t \) is the degree of capital utilization and the cost of capacity utilization, in terms of current production, is \( C(U_t)\bar{K}_{t-1} \), where \( C(U_t) = U_t^{1+\varsigma}/(1 + \varsigma) \).

The investment-specific shock \( d_t = \log D_t \) follows the stochastic process:

\[ d_t = \rho_d d_{t-1} + \varepsilon_{dt}. \]

\( \varepsilon_{dt} \) and all the variables denoted with \( \varepsilon \) from now on are i.i.d. shocks.
Consumption and investment are in terms of a final good which is produced by competitive final good producers using the CES production function

\[ Y_t = \left( \int_0^1 Y_{jt}^{1+\mu_{pt}} \, dj \right)^{1+\mu_{pt}} \]

which employs a continuum of intermediate inputs. \( Y_{jt} \) is the quantity of input \( j \) employed and \( \mu_{pt} \) captures a time-varying elasticity of substitution across goods, where \( \log(1 + \mu_{pt}) = \log(1 + \mu_p) + m_{pt} \) and \( m_{pt} \) follows the process \( m_{pt} = \rho_p m_{pt-1} + \varepsilon_{pt} - \psi_p \varepsilon_{pt-1} \).

The production function for intermediate good \( j \) is

\[ Y_{jt} = (K_{jt})^\alpha (A_t L_{jt})^{1-\alpha}, \]

where \( K_{jt} \) and \( L_{jt} \) are, respectively, capital and labor services employed. The technology parameter \( a_t = \log(A_t) \) follows the process

\[ a_t = a_{t-1} + \varepsilon_{t-4} + (1 - L)T_t \]
\[ T_t = \rho T_{t-1} + v_t, \]

where \( \varepsilon_t \) is a news shock that is known to agents at time \( t \), but will be reflected in \( a_t \) at time \( t + 4 \) and the part \( T_t \) is a persistent, but temporary, surprise technology shock.

BLL (2013) treat explicitly the constant term in TFP growth by letting \( A_t = \Gamma e^{a_t} \), but calibrate \( \Gamma = 1 \).

Intermediate good prices are sticky with price adjustment as in Calvo (1983). Each period intermediate good firm \( j \) can freely set the nominal price \( P_{jt} \) with probability \( 1 - \theta_p \) and with probability \( \theta_p \) is forced to keep it equal to \( P_{jt-1} \). These events are purely idiosyncratic, so \( \theta_p \) is also the fraction of firms adjusting prices each period.

Labor services are supplied to intermediate good producers by competitive labor agencies that combine specialized labor of types in \([0, 1]\) using the technology

\[ N_t = \left( \int_0^1 N_{jt}^{1+\mu_{wt}} \, dj \right)^{1+\mu_{wt}}, \]

where \( \log(1 + \mu_{wt}) = \log(1 + \mu_w) + m_{wt} \) and \( m_{wt} \) follows the process \( m_{wt} = \rho_w m_{wt-1} + \varepsilon_{wt} - \psi_w \varepsilon_{wt-1} \).

The presence of differentiated labor introduces monopolistic competition in wage setting as in Erceg, Henderson and Levin (2000). Specialized labor wages are also sticky and set by the household. For each type of labor \( j \), the household can freely set the price \( W_{jt} \) with probability \( 1 - \theta_w \) and has to keep it equal to \( W_{jt-1} \) with probability \( \theta_w \).

Market clearing in the final good market requires

\[ C_t + I_t + C(U_t)K_{t-1} + G_t = Y_t. \]
Market clearing in the market for labor services requires \( \int L_{jt}dj = N_t \).

Government spending is set as a fraction of output and the ratio of government spending to output is \( G_t/Y_t = \psi + g_t \), where \( g_t \) follows the stochastic process
\[
g_t = \rho g_{t-1} + \varepsilon_{gt}.
\]

Monetary policy follows the interest rate rule
\[
r_t = \rho_r r_{t-1} + (1 - \rho_r)(\gamma_n \pi_t + \gamma_g \hat{y}_t) + q_t,
\]
where \( r_t = \log R_t - \log R \) and \( \pi_t = \log P_t - \log P_{t-1} - \pi \), \( \pi \) is the inflation target, \( \hat{y}_t \) is defined below and \( q_t \) follows the process
\[
q_t = \rho q_{t-1} + \varepsilon_{qt}.
\]

The model is solved and a log-linear approximation around a deterministic steady-state is computed.

Given that TFP is non-stationary, some variables need to be normalized to ensure stationarity. We define \( \hat{c}_t \) as
\[
\hat{c}_t = \log(C_t/A_t) - \log(C/A),
\]
where \( C/A \) denotes the value of \( C_t/A_t \) in the deterministic version of the model in which \( A_t \) grows at the constant growth rate \( \Gamma \). Analogous definitions apply to the quantities \( \hat{y}_t, \hat{k}_t, \hat{\lambda}_t, \hat{\phi}_t \).

The quantities \( N_t \) and \( U_t \) are already stationary, so \( n_t = \log N_t - \log N \), and similarly for \( u_t \). For nominal variables, it is necessary to take care of non-stationarity in the price level, so: \( \hat{w}_t = \log(W_t/(A_tP_t)) - \log(W/(AP)) \), \( \hat{r}_k = \log(R^k_t/P_t) - \log(R^k/P) \), \( m_t = \log(M_t/P_t) - \log(M/P) \), \( r_t = \log(R_t - \log R \), \( \pi_t = \log(P_t/P_{t-1}) - \pi \).

Finally, for the Lagrange multipliers: \( \hat{\lambda}_t = \log(\Lambda_t A_t) - \log(\Lambda A), \hat{\phi}_t = \log(\Phi_t A_t/P_t) - \log(\Phi A/P) \). \( \Phi_t \) is the Lagrange multiplier on the capital accumulation constraint. The hat is only used for variables normalized by \( A_t \).

The first order conditions can be log-linearized to yield
\[
\hat{\lambda}_t = \begin{array}{c}
\frac{h^2 \beta \Gamma}{(\Gamma - \beta)(\Gamma - h)} E_t[\hat{c}_{t+1}] - \frac{\Gamma^2 + h^2 \beta}{(\Gamma - \beta)(\Gamma - h)} \hat{c}_t + \frac{h \Gamma}{(\Gamma - \beta)(\Gamma - h)} \hat{c}_{t-1} + \\
\frac{h \beta \Gamma}{(\Gamma - \beta)(\Gamma - h)} E_t[\Delta a_{t+1}] - \frac{h \Gamma}{(\Gamma - \beta)(\Gamma - h)} \Delta a_t \\
\end{array}
\]
\[
\hat{\lambda}_t = r_t + E_t[\hat{\lambda}_{t+1} - \Delta a_{t+1} - \pi_{t+1}]
\]
\[
\hat{\phi}_t = (1 - \delta) \beta \Gamma^{-1} E_t[\hat{\phi}_{t+1} - \Delta a_{t+1}] + (1 - (1 - \delta) \beta \Gamma^{-1}) E_t[\hat{\lambda}_{t+1} - \Delta a_{t+1} + r_{t+1}^k]
\]
\[ \hat{\lambda}_t = \phi_t + d_t - \chi \Gamma^2 (\hat{\nu}_t - \hat{\nu}_{t-1} + \Delta a_t) + \beta \chi \Gamma^2 E_t (\hat{\nu}_{t+1} - \hat{\nu}_t + \Delta a_{t+1}) \]

\[ r^k_t = \zeta u_t \]

\[ m_t = \alpha r^k_t + (1 - \alpha) \hat{w}_t \]

\[ r^k_t = \hat{w}_t - \hat{k}_t + n_t \]

Log-linearizing the accumulation equation for capital and the equation for capacity utilization, yields

\[ \hat{k}_t = u_t + \hat{k}_{t-1} - \Delta a_t \]

\[ \hat{k}_t = (1 - \delta) \Gamma^{-1} \left( \hat{k}_t - \Delta a_t \right) + (1 - (1 - \delta) \Gamma^{-1}) d_t + \hat{\nu}_t. \]

Approximating and aggregating the intermediate goods production function over producers and using the final good production function yields

\[ \hat{y}_t = \alpha \hat{k}_t + (1 - \alpha) n_t \]

Market clearing in the final good market yields

\[ (1 - \psi) \hat{y}_t = \frac{C}{Y} \hat{c}_t + \frac{I}{Y} \hat{\nu}_t + \frac{R^k K}{PY} u_t + g_t \]

\( C/Y, I/Y \) and \( R^k K/(PY) \) are all equilibrium ratios in the deterministic version of the model in which \( A_t \) grows at the constant rate \( \Gamma \).

Aggregating individual optimality conditions for price setters yields the Phillips curve

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa m_t + \kappa m_{pt} \]

where \( \kappa = (1 - \theta_p \beta)(1 - \theta_p)/\theta_p \).

Finally, aggregating individual optimality conditions for wage setters yields

\[ \hat{w}_t = \frac{1}{1 + \beta} \hat{w}_{t-1} + \frac{\beta}{1 + \beta} E_t \hat{w}_{t+1} - \frac{1}{1 + \beta} (\pi_t + \Delta a_t) + \frac{\beta}{1 + \beta} E_t (\pi_{t+1} + \Delta a_{t+1}) - \kappa_w \left( \hat{w}_t - \zeta n_t + \lambda_t + \kappa_w m_{wt} \right) \]
where \( \kappa_w = \frac{(1-\theta_w \beta)(1-\theta_w)}{\theta_w (1+\beta) \left( 1+\zeta \left( 1+\frac{1}{\mu_w} \right) \right)} \).

The log-linear model is estimated using Bayesian methods. Some parameters were calibrated using the mean values estimated in BLL. Table 4 reports the calibrated parameters.

Variables used in the estimation are the growth rates of output, consumption, investment and real wages, hours, the inflation rate and the federal funds rate (for details, see Appendix B). The choice of priors is very similar to the one used by BLL. Exception is made for the AR coefficients of the shocks, assumed here to be Normal with mean equal to 0 and standard deviation equal to 0.5 (0.4 for the coefficient \( \rho \) related to the transitory technology component) and for \( \sigma_d \) assumed here to be distributed as an Inverse Gamma with mean equal to 5 and standard deviation equal to 1.5.

We use an adaptive MCMC random walk Metropolis-Hastings algorithm (Haario et al. (2001)) to obtain the posterior distribution. Table 5 summarizes the priors and the posterior estimates of the parameters.
Appendix B: Data and data treatment

The data set includes US quarterly data on Total Factor Productivity, real per-capita GDP, real per-capita consumption of non-durables and services, real per-capita investment, real wages, per-capita hours worked, the federal funds rate and the inflation rate. The time span is 1954Q3-2015Q2, so that we have 243 time observations. TFP data are taken from the website of the Federal reserve Bank of San Francisco. The series is adjusted for capital utilization. Since data are provided in quarter-on-quarter growth rates, we took the cumulated sum to get level data. All other data are taken from the FRED data base. The original GDP series is real GDP in billions of chained 2009 dollars. Consumption is obtained as the sum of nominal personal consumption expenditures for services, divided by its implicit price deflator, and nominal personal consumption expenditures for nondurable goods, divided by its implicit price deflator. Investment is the sum of nominal private fixed investment, divided by its implicit price deflator, and nominal personal consumption expenditures for durable goods, divided by its implicit price deflator. The real wage is obtained from the BLS series “nonfarm business sector: compensation per hour”, divided by the GDP deflator. Hours worked are the BLS series named “nonfarm business sector: hours of all persons”. We divided GDP, consumption, investment and hours by civilian noninstitutional population (aged 16 years or more) to get per-capita figures, took the logs and multiplied by 100 so that the numbers appearing on the vertical axis are quarter-on-quarter variations expressed in percentage points. The federal funds rate is the monthly effective federal funds rate; we averaged monthly figures to get quarterly frequency and transformed the data to get quarterly rates in percentage points \((25\log(1 + r_t/100))\). Inflation is the first difference of the log of the GDP implicit price deflator multiplied by 100 to get figures expressed in percentage points.

Tables and Figures

<table>
<thead>
<tr>
<th>Horizon</th>
<th>0</th>
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<th>4</th>
<th>16</th>
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<td>(y_t), median estimate</td>
<td>0.00</td>
<td>0.10</td>
<td>0.12</td>
<td>0.12</td>
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<td>(y_t), true</td>
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<td>0.11</td>
<td>0.12</td>
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<td>(r_t), true</td>
<td>0.86</td>
<td>0.48</td>
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Table 1: Fraction of forecast error variance accounted for by the monetary policy shock in the empirical simulation of Example 1.
Table 2: The measure of informational deficiency $\delta$ for Example 1.

<table>
<thead>
<tr>
<th>Shocks of interest</th>
<th>$\delta(1)$</th>
<th>$\delta(4)$</th>
<th>$\delta(1000)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand shock, $d_t$</td>
<td>0.8904</td>
<td>0.8889</td>
<td>0.8889</td>
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<tr>
<td>Monetary shock $v_t$</td>
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<td>0.0000</td>
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Table 3: The measure of informational deficiency $\delta$ for Example 2.

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<th>Shocks of interest</th>
<th>$\delta(1)$</th>
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<tr>
<td>Shock $\varepsilon_t$</td>
<td>0.0347</td>
<td>0.0344</td>
<td>0.0342</td>
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<tr>
<td>Shock $d_t$</td>
<td>0.9732</td>
<td>0.9687</td>
<td>0.9653</td>
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<tr>
<td>Shock $e_t$</td>
<td>0.4891</td>
<td>0.2558</td>
<td>0.0899</td>
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</table>
Table 4: Calibrated parameters. We use the posterior mean values estimated by BLL.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\zeta$ (elasticity of $k$ utilization)</td>
<td>2.07</td>
</tr>
<tr>
<td>$\chi$ ($I$ adj. cost)</td>
<td>5.5</td>
</tr>
<tr>
<td>$h$ (habit persistence)</td>
<td>0.53</td>
</tr>
<tr>
<td>$\varsigma$ (inverse Frish elast.)</td>
<td>3.98</td>
</tr>
<tr>
<td>$\theta_w$ ($W$ stickiness)</td>
<td>0.87</td>
</tr>
<tr>
<td>$\theta_p$ ($P$ stickiness)</td>
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<tr>
<td>$\gamma_\pi$ ($\pi$ in Taylor rule)</td>
<td>1.003</td>
</tr>
<tr>
<td>$\gamma_y$ ($Y$ gap in Taylor rule)</td>
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<td>$\mu_p$ (SS $P$ markup)</td>
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<tr>
<td>$\mu_w$ (SS $W$ markup)</td>
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<tr>
<td>$\alpha$ (coeff. in prod. function)</td>
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<td>$\Gamma$ (TFP growth)</td>
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<tr>
<td>$\psi$ (G/Y)</td>
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<td>$\delta$ ($K$ depreciation)</td>
<td>0.025</td>
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<tr>
<td>$\beta$ (discount factor)</td>
<td>0.99</td>
</tr>
<tr>
<td>Parameter</td>
<td>Prior</td>
</tr>
<tr>
<td>----------------------------</td>
<td>------------</td>
</tr>
<tr>
<td>( \rho_r ) (i smoothing)</td>
<td>( Beta(0.5, 0.2) )</td>
</tr>
<tr>
<td>( \rho ) (temp. technology)</td>
<td>( \mathcal{N}(0.0, 0.4) )</td>
</tr>
<tr>
<td>( \rho_q ) (monetary)</td>
<td>( \mathcal{N}(0.0, 0.5) )</td>
</tr>
<tr>
<td>( \rho_d ) (I specific)</td>
<td>( \mathcal{N}(0.0, 0.5) )</td>
</tr>
<tr>
<td>( \rho_p ) (I specific)</td>
<td>( \mathcal{N}(0.0, 0.5) )</td>
</tr>
<tr>
<td>( \rho_w ) (W markup)</td>
<td>( \mathcal{N}(0.0, 0.5) )</td>
</tr>
<tr>
<td>( \rho_g ) (G)</td>
<td>( \mathcal{N}(0.0, 0.5) )</td>
</tr>
<tr>
<td>( \psi_p ) (MA in P mkup)</td>
<td>( Beta(0.5, 0.2) )</td>
</tr>
<tr>
<td>( \psi_w ) (MA in W mkup)</td>
<td>( Beta(0.5, 0.2) )</td>
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<tr>
<td>( \sigma_t ) (permanent tech.)</td>
<td>( \Gamma(0.5, 1.0) )</td>
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<tr>
<td>( \sigma_v ) (temporary tech.)</td>
<td>( \Gamma(1.0, 1.0) )</td>
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<tr>
<td>( \sigma_q ) (monetary)</td>
<td>( \Gamma(0.15, 1.0) )</td>
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<td>( \sigma_d ) (I specific)</td>
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<td>( \sigma_p ) (p markup)</td>
<td>( \Gamma(0.15, 1.0) )</td>
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<tr>
<td>( \sigma_w ) (w markup)</td>
<td>( \Gamma(0.15, 1.0) )</td>
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<tr>
<td>( \sigma_g ) (gov exp.)</td>
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<tr>
<td>Posterior value at mean</td>
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Table 5: Parameter estimates - mean. In brackets, the 5% and the 95% percentile of the posterior distribution.
<table>
<thead>
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<th>specification</th>
<th>news</th>
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<th>wage mkup</th>
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<th>mon. pol.</th>
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<td>S1</td>
<td>0.298</td>
<td>0.143</td>
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<td>S2</td>
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<td>0.175</td>
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<td>0.101</td>
<td>0.132</td>
<td>0.000</td>
<td>0.004</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 6: The measure of informational deficiency $\delta_i$, for each shock $i = 1, \ldots, 7$, for the VAR specifications S1-S10 listed in Section 3.

<table>
<thead>
<tr>
<th>specification</th>
<th>$\delta^1$</th>
<th>$\delta^2$</th>
<th>$\delta^4$</th>
<th>$\delta^{12}$</th>
<th>$\delta^{1000}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>S3</td>
<td>0.3349</td>
<td>0.0213</td>
<td>0.0204</td>
<td>0.0135</td>
<td>0.0072</td>
</tr>
<tr>
<td>S4</td>
<td>0.3332</td>
<td>0.0211</td>
<td>0.0198</td>
<td>0.0127</td>
<td>0.0062</td>
</tr>
<tr>
<td>S9</td>
<td>0.3220</td>
<td>0.0148</td>
<td>0.0139</td>
<td>0.0069</td>
<td>0.0003</td>
</tr>
<tr>
<td>S10</td>
<td>0.3178</td>
<td>0.0140</td>
<td>0.0132</td>
<td>0.0067</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Table 7: The measure of informational deficiency $\delta^K$, $K = 1, 2, 4, 12, 1000$, for the news shock, specifications S3, S4, S9, S10 in Section 3.
Figure 1: Impulse response functions of model (8) with $\alpha = 3$, $\gamma = 0.4$, $\beta = 1$ and standard normal shocks. Red solid lines: true impulse response functions. Black dashed lines: median of estimated impulse response functions. Grey area contains the 90% confidence interval computed from the Monte Carlo simulations. Lower panels report the distributions of the correlation coefficients between the estimated shocks and the true shocks (demand shock, $d_t$: left column; monetary policy shock, $v_t$: right column).
Figure 2: Impulse response functions of model (12) with $\beta = 0.99$, $\alpha = 0.5$, $\gamma = 20$, $\theta = 0.5$ and standard normal shocks. Red solid lines: true impulse response functions. Black dashed lines: median of estimated impulse response functions. Grey area contains the 90% confidence interval computed from the Monte Carlo simulations. Lower panels report the distributions of the correlation coefficients between the estimated shocks and the true shocks ($\varepsilon_t$, technology shock: left column; $d_t$, temporary shock to prices: right column).
Figure 3: Impulse responses of the Bayesian VAR with US data, for specification S1 (TFP and investment, top panels) and specification S2 (TFP and consumption, bottom panels). The dashed lines are the theoretical impulse response functions. The black solid lines are averages of the posterior distribution (500 draws). The dark gray and light gray areas are the 68% and 90% confidence bands.
Figure 4: Impulse responses of the Bayesian VAR, specification S3, estimated with artificial data generated from the economic model. The blue dashed lines are the theoretical impulse response functions. The red thick solid lines are obtained with 5000 time observations and 12 lags. The green dotted lines are obtained with 5000 time observations and 4 lags. The black thin solid lines are obtained with 243 time observations and 4 lags. The dark gray and light gray areas are the 68% and 90% confidence bands obtained with 4 lags and 243 observations.
Figure 5: Impulse responses of the Bayesian VAR, specification S10, estimated with artificial data generated from the economic model. The blue dashed lines are the theoretical impulse response functions. The red thick solid lines are obtained with 5000 time observations and 12 lags. The green dotted lines are obtained with 5000 time observations and 4 lags. The black thin solid lines are obtained with 243 time observations and 4 lags. The dark gray and light gray areas are the 68% and 90% confidence bands obtained with 4 lags and 243 observations.
Figure 6: Impulse responses of the Bayesian VAR, specification S3, estimated with real US data. The dashed lines are the theoretical impulse response functions. The black solid lines are the empirical impulse response functions (averages of the posterior distribution, 500 draws). The dark gray and light gray areas are the 68% and 90% confidence bands.
Figure 7: Impulse responses of the Bayesian VAR, specification S10, estimated with real US data. The dashed lines are the theoretical impulse response functions. The black solid lines are the empirical impulse response functions (averages of the posterior distribution, 500 draws). The dark gray and light gray areas are the 68% and 90% confidence bands.
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