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Operational–risk Dependencies and the Determination of Risk Capital

Stefan Mittnik^{a,b,d,*}, Sandra Paterlini^{b,d}, Tina Yener^{c,d}

^aChair of Financial Econometrics, Department of Statistics, University of Munich, Germany

^bDepartment of Economics, CEFIN and RECent, University of Modena and Reggio E., Italy

^cTreasury/Risk Department, Linde AG, Munich, Germany

^dCenter for Quantitative Risk Analysis, University of Munich, Germany

Abstract

With the advent of Basel II, risk–capital provisions need to also account for operational risk. The specification of dependence structures and the assessment of their effects on aggregate risk–capital are still open issues in modeling operational risk. In this paper, we investigate the potential consequences of adopting the restrictive Basel’s Loss Distribution Approach (LDA), as compared to strategies that take dependencies explicitly into account. Drawing on a real–world database, we fit alternative dependence structures, using parametric copulas and nonparametric tail–dependence coefficients, and discuss the implications on the estimation of aggregate risk capital.

We find that risk–capital estimates may increase relative to that derived for the LDA when accounting explicitly for the presence of dependencies. This phenomenon is not only be due to the (fitted) characteristics of the data, but also arise from the specific Monte Carlo setup in simulation–based risk–capital analysis.

Keywords: Copula, Nonparametric Tail Dependence, Basel II, Loss Distribution Approach, Value–at–Risk, Subadditivity

JEL: C14, C15, G10, G21

*Corresponding author: Stefan Mittnik, Chair of Financial Econometrics, Address: Akademiestr. 1/I, 80799 Munich, Germany. Tel. +49 (0) 8921803224, Fax. +49 (0) 8921805044, Email: finmetrics@stat.uni-muenchen.de

1. Introduction

The operational risk of financial institutions has received increasing attention during recent years, mainly because of new transparency requirements in financial reporting, but also because of the magnitude of several, highly-publicized operational-loss cases. For example, only in 2008, the ten largest operational losses exceeded 1.8 billion U.S. dollars, with a maximum loss of 50 billion from the Madoff fraud, followed by 8.4 billion losses for Wells Fargo & Co. The magnitude of these losses supports the notion that a financial institution's capital requirements for operational losses may well be larger than those for market risk (de Fontnouvelle et al., 2003). This concern emphasizes the importance of implementing reliable procedures for estimating appropriate capital buffers for operational losses.

The Basel committee (Basel Committee on Banking Supervision, 2006) formally defines operational risk as “the risk of loss resulting from inadequate or failed internal processes, people and systems or from external events.” Although, since the introduction of the Basel II accord in 2004, operational risk has become a relevant factor when determining a financial institution's overall capital requirements, quantifying and modeling operational risk still poses numerous challenges. One major challenge is to account for the dependencies among the individual operational-risk components. Ideally, risk-capital assessment takes such dependencies adequately into account, in order to derive realistic charges for regulatory capital.

The Loss Distribution Approach (LDA), the most popular method within the Basel II Advanced Measurement Approach for operational risk, abstains from modeling dependencies explicitly. Instead, the aggregate Value-at-Risk (VaR) for operational risk is obtained by simply adding up the individual VaRs derived for each risk component. Intuitively, by summing up the individual capital charges one expects to determine a kind of worst-case capital buffer. However, the non-coherence, specifically, the potential superadditivity of the VaR measure, as well as particular distributional properties of operational-loss data may, in fact, have the opposite effect: By departing from the summation rule and accounting explicitly for dependencies, aggregate risk-capital estimates may increase rather than decrease. The theoretical possibility of this to happen has been mentioned before (see, for example, Embrechts et al., 2002), but the practical relevance and potential magnitude, when dealing with real-world operational-loss data, remains to be investigated.

This gap is the focus of this paper. Drawing on a real-world database of opera-

tional losses,¹ we analyze the dependence structure of monthly aggregate losses using alternative modeling strategies, namely, parametric copula fitting and nonparametric tail-dependence estimation. Then, by means of Monte Carlo simulation, we assess the changes in risk-capital estimates when imposing empirically determined dependence structures.

Our empirical findings suggest that, when taking dependence structures into account, risk-capital estimates may be up to 30% higher than those obtained under the simple summation scheme of the LDA. Therefore, accounting explicitly for dependencies may lead to more realistic, but not necessarily lower estimates for regulatory capital. The increase may not only be due to the superadditivity problem, but also arise from computational settings. Specifically, the higher the number of replications in the Monte-Carlo simulation, the lower the variation of risk-capital estimates. An excessively high aggregate risk-capital estimate may, therefore, not only be due to strong dependencies, but also due to an insufficient number of Monte Carlo replications. To disentangle the effects of these two sources, we compute asymptotic bounds for VaR-estimates obtained from imposing restrictions on the underlying copula.

The paper is organized as follows. Section 2 briefly discusses the aggregation problem in multivariate operational-risk modeling. The empirical analysis, based on loss data collected in the DIPO database, is presented in Section 3. Section 4 explores how aggregate risk-capital estimation can be affected by the treatment of dependencies among risk components. Section 5 concludes.

2. Operational Risk and the Aggregation Problem

According to the LDA, an institution's business operations are classified into—at least—eight business lines, each being exposed to seven types of loss-events. This gives rise to $7 \times 8 = 56$ event-type/business-line combinations, as illustrated in Table 1. Given the heterogeneous nature of these combinations, VaR-calculations require a case-by-case analysis for each of the 56 risk categories. To obtain the aggregate capital charge, however, the individual VaRs need to be combined into a single, overall operational

¹The data are from the *Database Italiano delle Perdite Operative* (DIPO) association, a consortium of 31 Italian banking groups collecting operational-loss data from more than 200 legal entities (see <http://www.dipo-operationalrisk.it>).

VaR-figure, i.e.,

$$\text{VaR}_\alpha(L) = \text{VaR}_\alpha\left(\sum_{i=1}^{56} L_i\right), \quad (1)$$

where $L = \sum_{i=1}^{56} L_i \sim F_L$ refers to aggregate losses; and

$$\text{VaR}_\alpha = \inf\{l \in \mathbb{R} : \Pr[L > l] \leq 1 - \alpha\} = \inf\{l \in \mathbb{R} : F_L(l) \geq \alpha\}$$

denotes the VaR at confidence level $(100 \times \alpha)\%$ or, in other words, the $\alpha\%$ quantile of the aggregate loss distribution. Clearly, risk-capital estimates derived from (1) will be affected by dependencies among the 56 risk categories. Therefore, a reliable derivation of operational-risk capital along the lines of (1) requires the specification of dependence structures.

Table 1: Business-line/event-type matrix.

		Business Lines							
		Corporate Finance	Trading & Sales	Retail Banking	Commercial Banking	Payment & Settlement	Agency Services	Asset Management	Retail Brokerage
Event Types	Internal Fraud	L_1	L_8	
	External Fraud	
	Employment Practices & Workplace Safety	
	Clients, Products & Business Practices								
	Damage to Physical Assets	
	Business Disruption & System Failures	
	Execution, Delivery & Process Management	L_{49}	L_{56}	

Rather than following (1), the LDA calculates the total risk capital (TRC) for operational losses by summing up the individual component-VaRs, i.e.,

$$\text{TRC} = \sum_{i=1}^{56} \text{VaR}_\alpha(L_i), \quad (2)$$

with the confidence level set to $\alpha = 0.999$.

Only when a financial institution fulfills certain qualifying conditions, operational–risk aggregation may take dependencies explicitly into account. In this case, the crucial question is: To what extent will an explicit modeling of dependencies, that is, adopting (1) instead of (2), affect aggregate risk–capital estimates? More specifically, will the additional effort, when modeling along the lines of (1), not only be compensated by a better understanding of the institution’s risk exposures, but also by a reduction in regulatory capital?

If L_i and L_j are comonotonic,² we have (see Embrechts et al., 2003b; McNeil et al., 2005)

$$\text{VaR}_\alpha^{\text{co}}(L_i + L_j) = \text{VaR}_\alpha(L_i) + \text{VaR}_\alpha(L_j) .$$

For the—in practice widely adopted—family of elliptical distributions and, thus, for the Gaussian case, comonotonicity translates into perfect positive correlation. In general, for elliptical distributions we have

$$\text{VaR}_\alpha(L_i + L_j) \leq \text{VaR}_\alpha(L_i) + \text{VaR}_\alpha(L_j) ,$$

with strict inequality holding in non-degenerate cases. Therefore, the LDA calculation according to (2) represents a worst–case scenario for aggregate risk. The fact that one may derive more realistic and practically relevant rather than worst–case and, presumably, higher TRC–estimates provides an incentive for modeling dependencies explicitly. In fact, several studies, such as Frachot et al. (2001), Chavez-Demoulin et al. (2006) and Chapelle et al. (2008), point out that the assumption of perfect positive dependence is an unduly strong and unrealistic restriction and show that modeling dependencies via empirical correlations can decrease TRC–charges by a factor of about 30%.

Outside the world of elliptical distributions, the presence of dependence structures may, indeed, increase TRC–charges, given the VaR–measure’s lack of subadditivity (Artzner et al., 1999); i.e., we may have

$$\text{VaR}_\alpha(L_i + L_j) > \text{VaR}_\alpha(L_i) + \text{VaR}_\alpha(L_j) .$$

²Comonotonicity represents an extreme case of perfect dependence, where one loss category is a deterministic function of another; i.e., $L_i = T(L_j)$ holds almost surely with T being a strictly increasing function.

In other words, efforts to reduce TRC-charges by specifying dependencies more appropriately may have the opposite effect and increase risk-capital estimates beyond the level prescribed by Basel's LDA.³

To quantify the potential of diversification among N risk components, we adopt the diversification measure

$$D_\alpha = \frac{\text{VaR}_\alpha(\sum_{i=1}^N L_i) - \sum_{i=1}^N \text{VaR}_\alpha(L_i)}{\sum_{i=1}^N \text{VaR}_\alpha(L_i)},$$

which is similar to that in Embrechts et al. (2009), but expressed in relative terms. Negative (positive) values of D_α indicate that, compared to simply adding individual VaRs, risk-capital estimates will decrease (increase) when accounting for dependencies. Considering only two risk components, i and j , and a 99.9% confidence level, the measure reduces to

$$D_{.999}^{i,j} = \frac{\text{VaR}_{.999}(L_i + L_j) - [\text{VaR}_{.999}(L_i) + \text{VaR}_{.999}(L_j)]}{\text{VaR}_{.999}(L_i) + \text{VaR}_{.999}(L_j)}.$$

In the following sections, we investigate whether an explicit modeling of dependencies generally affects aggregate risk-capital estimates, and whether working along the lines of (1) will save regulatory capital. The latter is of paramount importance, as there should be an incentive for financial institutions to better understand their risk structures and, thus, to adopt an explicit modeling approach. If, on the other hand, the naive, black-box approach (2)—in addition to being less burdensome and less costly—will tend to produce lower capital-charges, institutions may have an incentive to remain largely uninformed about the risks they are exposed to.

3. Empirical Analysis

3.1. The Data

Since 2003, the *Database Italiano delle Perdite Operative* (DIPO) association collects twice a year operational-loss data exceeding a reporting-threshold of 5,000 Euros. For all participating institutions, the definitions of gross loss, event type and business

³Relationships between superadditivity and properties of the underlying (joint and marginal) distributions are, for example, discussed in Embrechts et al. (2009).

line are identical. With 31 members (amounting to 209 different entities) of all sizes, the DIPO database represents about 75% of the Italian banking system in terms of gross income and operating costs.

Despite the substantial sample size, it turns out that a number of the 7×8 business–line/event–type cells are empty, reflecting that no losses have been recorded. For this reason, we conduct our analysis on a more aggregate level. To do so, we could aggregate across either the event–type or the business–line dimension. Both approaches can be found in the literature.⁴ Which of the two strategies is to be preferred depends on the objective of the analysis. For management purposes, it appears more reasonable to analyze business lines separately, in order to assess loss profiles and identify divisions suffering from negative developments. On the other hand, it seems more natural that events of the same type, such as fraud or storm losses, follow distributions with similar characteristics, so that statistical fitting might give better results than when aggregating over business lines. This last argument and the fact that both the number of events and the loss amounts are distributed more evenly across event types than across business lines (see Tables 2 and 3) are the reasons why we model along the event–type dimension in the following.

Empirical operational–risk analyses can be hampered by data features, such as non-stationarities and seasonalities (Embrechts et al., 2003a), which have to be accounted for in the modeling process. We do this by choosing an appropriate level of temporal aggregation, i.e., when deciding on using weekly, monthly or quarterly loss data. Although a weekly aggregation level leaves us with more data points, many of these are, however, equal to zero; moreover, seasonalities and loss clustering are rather prevalent. At a monthly frequency, we find that large losses tend to occur around the turn of the year and that there are clusters of months with many losses and high aggregate losses. Quarterly aggregation partially eliminates these features, but reduces the number of observations substantially. To balance the tradeoff between having a large sample size and working with data that are approximately independent and identically distributed, we use monthly data—i.e., we aggregate all losses occurring within one calendar month—which gives rise to 60 observations per event–type.

⁴Moscadelli (2004), for example, groups by business line and El-Gamal et al. (2006) by event type.

Table 2: Distribution across event types. Period: Jan 2003 - Dec 2007.

Event type	% of events	Total loss (in %)
1 Internal Fraud	2.56	17.60
2 External Fraud	37.01	19.37
3 Employment Practices & Workplace Safety	5.93	6.87
4 Clients, Products & Business Practices	28.69	37.31
5 Damage to Physical Assets	2.65	1.34
6 Business Disruption & System Failures	1.16	1.12
7 Execution, Delivery & Process Management	21.97	16.39

Table 3: Distribution across business lines. Period: Jan 2003 - Dec 2007.

Business line	% of events	Total loss (in %)
1 Corporate Finance	0.14	0.17
2 Trading & Sales	5.08	11.51
3 Retail Banking	62.46	45.25
4 Commercial Banking	7.61	23.64
5 Payment & Settlement	0.69	0.42
6 Agency Services	0.62	0.85
7 Asset Management	0.68	0.59
8 Retail Brokerage	22.73	17.57

3.2. Correlation

The most common approach to capturing dependence structures is to estimate linear (Pearson) correlations. Linear correlation estimates for all seven event types are reported in Table 4. The upper, right triangle in Table 4 displays estimates from the full sample, January 2003 – December 2007, the lower, left part those obtained from the subperiod January 2003 – April 2006. Looking at the full-sample estimates first, we observe that correlations may vary substantially across event-type combinations. Combination (2;5) (External Fraud; Damage to Physical Assets), for example, shows little correlation, whereas Combination (3;4) (Employment Practices & Workplace Safety; Clients, Products & Business Practices) exhibits stronger correlation with $\rho = 0.528$.⁵ This suggests that event-type-specific characteristics need to be allowed for when assessing dependence patterns in operational-risk data.

In the following, we will restrict our attention to event-type Combinations (2;5) and (3;4), as they cover a range of possible dependence structures. This does not imply that our conclusions are specific to these combinations. They may rather cover the wide spectrum of loss-data characteristics encountered in practice.

Turning to the correlation estimates from the smaller subsample and comparing upper and lower triangles in Table 4, we observe that some correlation estimates are rather unstable—in financial risk analysis, a widely observed deficiency of linear correlation estimates. This instability is illustrated in Figure 1, showing scatter plots of aggregate losses for the two event-type combinations (upper and lower panels) and the two sample sizes (left and right panels). The presence of one or two extreme observations, encircled in the full-sample plot (left panel), or their absence in the subsample (right panel) explains the variation in the correlation estimates. An even more drastic change in the linear correlation estimates results from elimination of the most extreme observation in

⁵This finding is in line with intuition. Event-types 3 and 4 are both affected by management practices, whereas External Fraud and Damage to Physical Assets tend to be attributable to a broad range of causes.

Table 4: Estimated linear (Pearson) correlations for full-sample period 01/2003–12/2007 (upper triangle) and subsample 01/2003–04/2006 (lower triangle)

	ET 1	ET 2	ET 3	ET 4	ET 5	ET 6	ET 7
ET 1	1	0.045	0.221	0.046	0.010	0.085	0.223
ET 2	0.062	1	0.162	0.161	0.026	-0.066	0.052
ET 3	0.126	0.283	1	0.528	0.295	0.182	0.443
ET 4	-0.028	0.242	0.588	1	0.143	0.063	0.201
ET 5	-0.012	-0.114	0.290	0.177	1	-0.046	-0.050
ET 6	0.030	-0.039	0.151	0.012	-0.053	1	0.065
ET 7	0.155	0.063	0.365	0.202	-0.104	0.045	1

the smaller sample for Combination (3;4), causing the correlation estimate to decrease from 0.588 to 0.323.

Table 5: Estimated Kendall rank correlations for full-sample period 01/2003–12/2007 (upper triangle) and subsample 01/2003–04/2006 (lower triangle).

	ET 1	ET 2	ET 3	ET 4	ET 5	ET 6	ET 7
ET 1	1	0.135	0.248	0.214	0.142	0.105	0.190
ET 2	0.097	1	0.148	0.211	0.005	-0.024	0.111
ET 3	0.064	0.192	1	0.510	0.260	0.207	0.183
ET 4	0.049	0.218	0.451	1	0.307	0.313	0.072
ET 5	0.046	-0.005	0.162	0.356	1	0.052	-0.033
ET 6	0.041	0.097	0.151	0.280	0.051	1	0.172
ET 7	0.133	0.082	0.090	-0.044	-0.139	0.139	1

In view of the volatility of our Pearson-correlation estimates, rank-based correlations may be preferable, as they tend to yield more robust estimates. They are derived from ranked observations and, thus, are less affected by extremes. Tables 5 and 6, constructed in the same way as Table 4, report estimates for Kendall’s τ and Spearman’s ρ , respectively. The estimates for these statistics reveal that not only the size-based

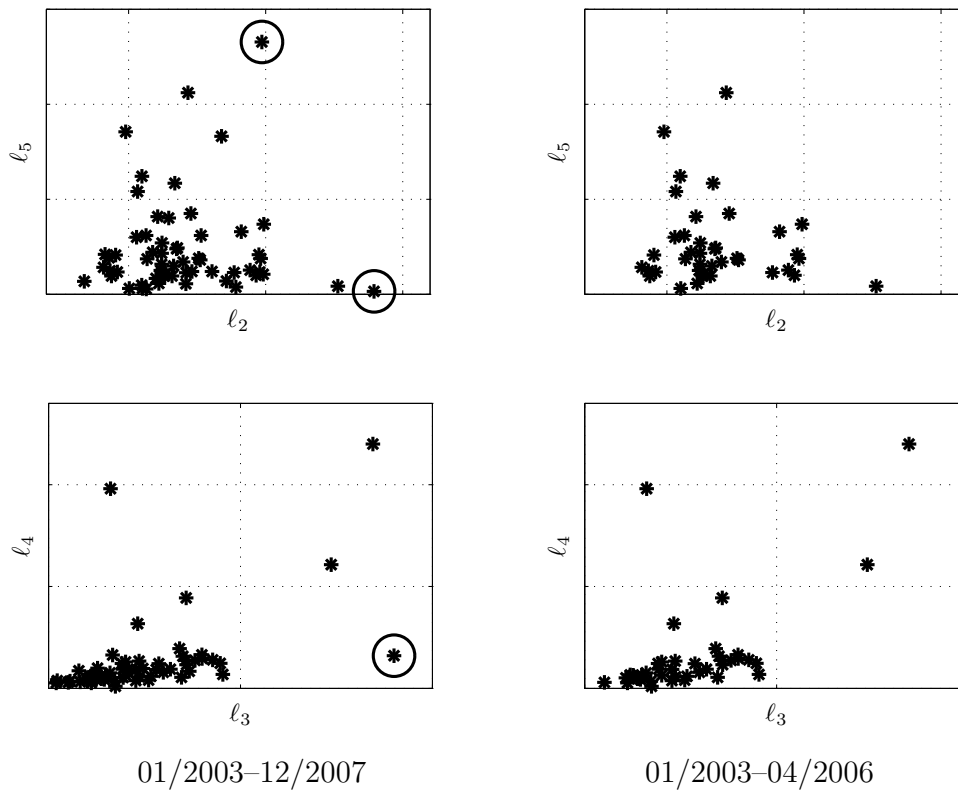


Figure 1: Scatter plots of aggregate losses for event-type Combinations (2;5) (top) and (3;4) (bottom) for the full sample (left) and the subsample (right).

but also the rank-based correlation estimates are quite unstable.

These findings suggest that, in the context of operational losses, both linear and rank correlations tend to be inappropriate measures of dependence. One reason for this can be due to the fact that—just like Pearson’s correlation—rank correlations are still too rigid to handle complex dependence structures. The copula approach, discussed next, offers more flexibility in that regard.

Table 6: Estimated Spearman rank correlations for full-sample period 01/2003–12/2007 (upper triangle) and subsample 01/2003–04/2006 (lower triangle).

	ET 1	ET 2	ET 3	ET 4	ET 5	ET 6	ET 7
ET 1	1	0.197	0.343	0.312	0.223	0.152	0.293
ET 2	0.157	1	0.202	0.293	0.001	-0.029	0.153
ET 3	0.057	0.276	1	0.695	0.385	0.320	0.257
ET 4	0.071	0.301	0.633	1	0.433	0.451	0.109
ET 5	0.096	-0.002	0.258	0.494	1	0.079	-0.043
ET 6	0.062	0.146	0.236	0.411	0.088	1	0.272
ET 7	0.191	0.112	0.125	-0.045	-0.202	0.226	1

3.3. Copulas

The copula approach provides a framework for handling more complex forms of dependence than conventional Pearson correlation, which restricts dependencies between two random variables to proportional variations that hold globally over the range of the data. Copulas offer more flexibility by allowing dependence patterns to vary as loss levels change.⁶

Given n loss types, L_1, \dots, L_n , with joint distribution function $F(\cdot, \dots, \cdot)$ and marginals F_1, \dots, F_n , a copula function, $C(\cdot, \dots, \cdot)$, “couples” the marginal distributions via Sklar (1959)

$$\Pr[L_1 \leq \ell_1, \dots, L_n \leq \ell_n] = F(\ell_1, \dots, \ell_n) = C(F_1(\ell_1), \dots, F_n(\ell_n)) . \quad (3)$$

A copula function captures the entire dependence structure between losses L_1, \dots, L_n , while being invariant under strictly increasing transformations of the marginals. This is a clear advantage over the Pearson correlation coefficient, which is affected by changes in the marginal distributions. The explicit representation of the copula function,

$$C(u_1, \dots, u_n) = F(F_1^{-1}(u_1), \dots, F_n^{-1}(u_n)) ,$$

⁶For a general introduction to copulas, see Nelsen (1999) and Cherubini et al. (2004).

with $u_1, \dots, u_n \sim \text{Unif}(0, 1)$, shows how to “extract” a copula from the joint distribution function, $F(\cdot, \dots, \cdot)$. Copulas may also be applied to survival functions, $\bar{F}(\ell) = \Pr[L > \ell] = 1 - F(\ell)$. For the joint survival function

$$\bar{F}(\ell_1, \ell_2) = \Pr[L_1 > \ell_1, L_2 > \ell_2] = 1 - F_1(\ell_1) - F_2(\ell_2) + F(\ell_1, \ell_2) ,$$

we obtain a relationship analogous to (3), i.e.,

$$\bar{F}(\ell_1, \ell_2) = \bar{C}(\bar{F}_1(\ell_1), \bar{F}_2(\ell_2)) ,$$

where

$$\bar{C}(u_1, u_2) = u_1 + u_2 - 1 + C(1 - u_1, 1 - u_2) \tag{4}$$

is the survival copula for L_1 and L_2 .

In our analyses, we consider copula families that are commonly used in practice. They include the Gaussian copula

$$C^\Phi(u_1, \dots, u_n) = \Phi_n(\Phi^{-1}(u_1), \dots, \Phi^{-1}(u_n); \rho) ,$$

where $\Phi_n(\cdot)$ denotes the multivariate cumulative distribution function (cdf) of the standard normal distribution with correlation matrix ρ ; and Φ is the univariate standard normal cdf. Another elliptic copula we consider is the Student- t copula

$$C^\Psi(u_1, \dots, u_n) = \Psi_n(\Psi^{-1}(u_1), \dots, \Psi^{-1}(u_n); \rho, \nu) ,$$

which is implied by the multivariate Student- t distribution with correlation matrix ρ and degrees-of-freedom parameter ν .

Archimedean copulas are alternatives to elliptic copulas. Below, we consider Gumbel and Clayton copulas, given by

$$C^G(u_1, \dots, u_n) = \exp \left\{ - \left(\sum_{i=1}^n (-\ln(u_i))^\alpha \right)^{1/\alpha} \right\}, \quad \alpha > 1$$

and

$$C^C(u_1, \dots, u_n) = \left(\sum_{i=1}^n u_i^{-\gamma} - n + 1 \right)^{-1/\gamma}, \quad \gamma > 0,$$

respectively.

The dependence structures implied by these copulas are illustrated in Figure 2, showing 5,000 random draws from bivariate copulas. Although, in all cases in Figure 2, Kendall's τ is set to $\tau = 0.6$, the resulting joint behavior of the components differs greatly across the copula types. This is mainly due to the copulas' *tail behavior* or, more specifically, the nature of their *tail-dependence*—a property that cannot be captured by a correlation measure.

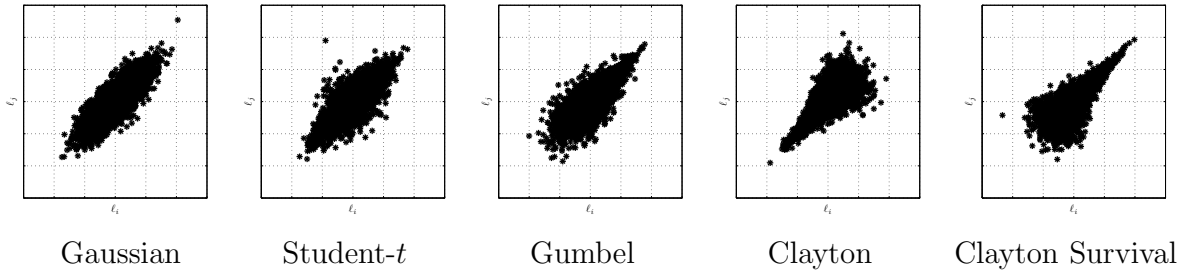


Figure 2: Dependence structures associated with different copula functions.

The coefficient of upper-tail dependence, denoted by λ_U , characterizes the joint behavior of the operational losses as the loss level, t , rises and is given by

$$\lambda_U = \lim_{t \rightarrow 1^-} \Pr[F_i(\ell_i) > t | F_j(\ell_j) > t].$$

It can also be expressed in terms of the copula of L_1 and L_2 via

$$\lambda_U = \lim_{t \rightarrow 1^-} \frac{1 - 2t + C(t, t)}{1 - t}. \quad (5)$$

The Gaussian copula implies a tail-dependence coefficient of zero. The Student- t copula is characterized by tail dependence in both the upper and the lower tail; i.e., the

data cluster symmetrically in the extremes of the distribution. Archimedean copulas allow for asymmetric dependence structures, with the Gumbel copula exhibiting upper-tail and the Clayton copula lower-tail dependence.

Table 7: Copulas and coefficients of upper-tail dependence.

Copula	Θ	λ_U
Gaussian	ρ	0
Student- t	(ρ, ν)	$2 - 2\Psi_n\left(\sqrt{\nu}\frac{\sqrt{1-\rho}}{\sqrt{1+\rho}}; \rho, \nu\right)$
Gumbel	α	$2 - 2^{1/\alpha}$
Gumbel Survival	α_S	0
Clayton	γ	0
Clayton Survival	γ_S	$2^{-1/\gamma_S}$

When modeling operational losses, only upper-tail dependence is of relevance. To still use copulas exhibiting only lower-tail dependence, we “flip” the copula via (4), to obtain the corresponding survival copula, which represents the “mirror image” of the original copula. For example, in the case of the Clayton copula, we obtain the Clayton survival copula, which exhibits upper-tail dependence, as shown in the far right scatter plot in Figure 2. The relationships between copula parameters, denoted by Θ , and the upper-tail-dependence coefficients are summarized in Table 7.

Depending on how the marginal distributions are treated, there are different ways of estimating copulas with maximum likelihood.⁷ Below, we do not estimate any parametric marginal distributions, but rather use empirical densities as inputs to the likelihood function.⁸ The rationale for this is that we do not want to influence the estimation of

⁷An overview of different approaches to copula estimation can be found in Cherubini et al. (2004).

⁸To be precise, we use a rescaled version of the cdf, which divides by $n + 1$, in order to avoid problems as u_i approaches unity; see Genest et al. (1995).

the dependence structure by a parametric restrictions on the marginals.

Table 8: Maximum-likelihood estimation for alternative parametric copulas for event-type Combination (2;5).

	$\hat{\Theta}$	$\hat{\lambda}_U$	$-\ell(\hat{\Theta})$	AIC	BIC	p -value \mathcal{A}_2	p -value \mathcal{A}_4
01/2003–12/2007							
Gaussian	-0.015	0.000	-0.005	1.989	8.178	0.640	0.619
Student- t	3.86	0.095	-0.945	0.110	6.299	0.798	0.606
Gumbel	1.015	0.020	-0.016	1.969	8.157	0.790	0.769
Gumbel Survival	1.000	0.000	0.000	2.000	8.189	0.697	0.743
Clayton	0.023	0.000	-0.009	1.982	8.171	0.792	0.775
Clayton Survival	0.002	0.000	-0.000	2.000	8.189	0.692	0.628
01/2003–04/2006							
Gaussian	-0.014	0.000	-0.003	1.994	7.372	0.7053	0.599
Student- t	7.51	0.027	-0.092	1.817	7.194	0.7592	0.645
Gumbel	1.00	0.000	0.000	2.001	7.378	0.6254	0.613
Gumbel Survival	1.01	0.000	-0.001	1.998	7.376	0.6943	0.670
Clayton	0.081	0.000	-0.068	1.864	7.242	0.8581	0.849
Clayton Survival	0.000	0.000	0.000	2.000	7.378	0.6583	0.648

The estimation results for the event-type Pair (2;5) are reported in Table 8. The upper (lower) part of Table 8 states the results for the full (first two thirds of the) sample. The first column shows the parameter estimates obtained by the semiparametric maximum-likelihood procedure. For the Gaussian copula, the estimated correlations are $\hat{\rho} = -0.015$, for the full sample, and $\hat{\rho} = -0.014$, for the shorter sample. These are

more stable estimates than those for the Pearson correlation (see Table 4). The second column reports the coefficients for upper-tail dependence as implied by the copula estimates and the relations reported in Table 7. Of the three copulas capable of capturing upper-tail dependence, only the Student- t yields a somewhat sizable value; the Gumbel suggests little and the Clayton survival no tail dependence.

The subsequent columns in Table 8 assess the goodness of fit: the negative log-likelihood (Column 3), followed by two information or model-selection criteria, the AIC and BIC.⁹ Both the AIC and BIC favor the Student- t copula for the full and the small sample, though their values are quite close for the small sample. The final two columns present p -values from two goodness-of-fit tests, which, according to the simulation studies by Berg (2009), seem to perform well: the \mathcal{A}_2 -test of Genest and Rémillard (2008) and the \mathcal{A}_4 -test (Genest et al., 2006). Both tests are based on the distances between the empirical and the fitted copulas which enter a Cramer-van-Mises statistic.¹⁰ None of the p -values derived from the bootstrap procedures allows us to reject any of the copulas under consideration.

Table 9 presents the estimation results for Combination (3;4). Here, for both samples, the AIC and BIC select copulas exhibiting substantial tail dependence, but the favored copula-type varies with the sample (the Student- t , with $\hat{\nu} = 0.40$, for the full and the Gumbel, with $\hat{\alpha} = 0.56$, for the partial sample). However, the values obtained for the information criteria are quite close. As with Pair (2;5), based on the goodness-

⁹The Akaike (AIC) and Bayesian Information Criteria (BIC) are given by

$$\begin{aligned} AIC &= -2\ln(\ell(\hat{\Theta})) + 2q \\ BIC &= -2\ln(\ell(\hat{\Theta})) + q\ln(T) \end{aligned}$$

where q refers to the number of estimated parameters and T denotes the sample size. The model associated with the lowest criterion value is favored by the data under investigation.

¹⁰We use the same numbers of replications as Berg (2009), namely, 10,000 for the parametric bootstrap and an additional 2,500 replications in the double bootstrap.

Table 9: Maximum–likelihood estimation results for alternative parametric copulas for event–type Combination (3;4).

	$\hat{\Theta}$	$\hat{\lambda}_U$	$-\ell(\hat{\Theta})$	AIC	BIC	p -value \mathcal{A}_2	p -value \mathcal{A}_4
	01/2003–12/2007						
Gaussian	0.719	0.000	-19.489	-36.977	-30.789	0.912	0.920
Student- t	6.618	0.402	-19.971	-37.941	-31.752	0.961	0.935
Gumbel	1.954	0.574	-18.386	-34.772	-28.583	0.285	0.499
Gumbel Survival	1.986	0.000	-19.443	-36.885	-30.696	0.859	0.828
Clayton	1.522	0.000	-17.065	-32.129	-25.940	0.103	0.228
Clayton Survival	1.398	0.609	-15.150	-28.300	-22.111	0.010	0.048
	01/2003–04/2006						
Gaussian	0.682	0.000	-10.593	-19.1864	-13.809	0.8452	0.9071
Student- t	5.865	0.390	-11.023	-20.0453	-14.668	0.9181	0.8362
Gumbel	1.910	0.562	-11.315	-20.6297	-15.252	0.6134	0.7293
Gumbel Survival	1.795	0.000	-9.630	-17.2595	-11.882	0.5594	0.8561
Clayton	1.193	0.000	-8.341	-14.6819	-9.304	0.0909	0.3117
Clayton Survival	1.494	0.629	-10.017	-18.0344	-12.657	0.1978	0.3357

of–fit tests, (with the exception of the Clayton survival copula for the full sample) it is not possible to reject any of the copulas entertained. Overall, however, we obtain superior fits for event–type Pair (3;4) than for Pair (2;5).

Figure 3 illustrates the fit of the copulas by comparing contour plots of empirical and fitted copulas. The left (right) panel is based on the full (partial) sample. Whereas for Pair (2;5) (upper part) all fitted copula contours are close, they vary for (3;4) (lower

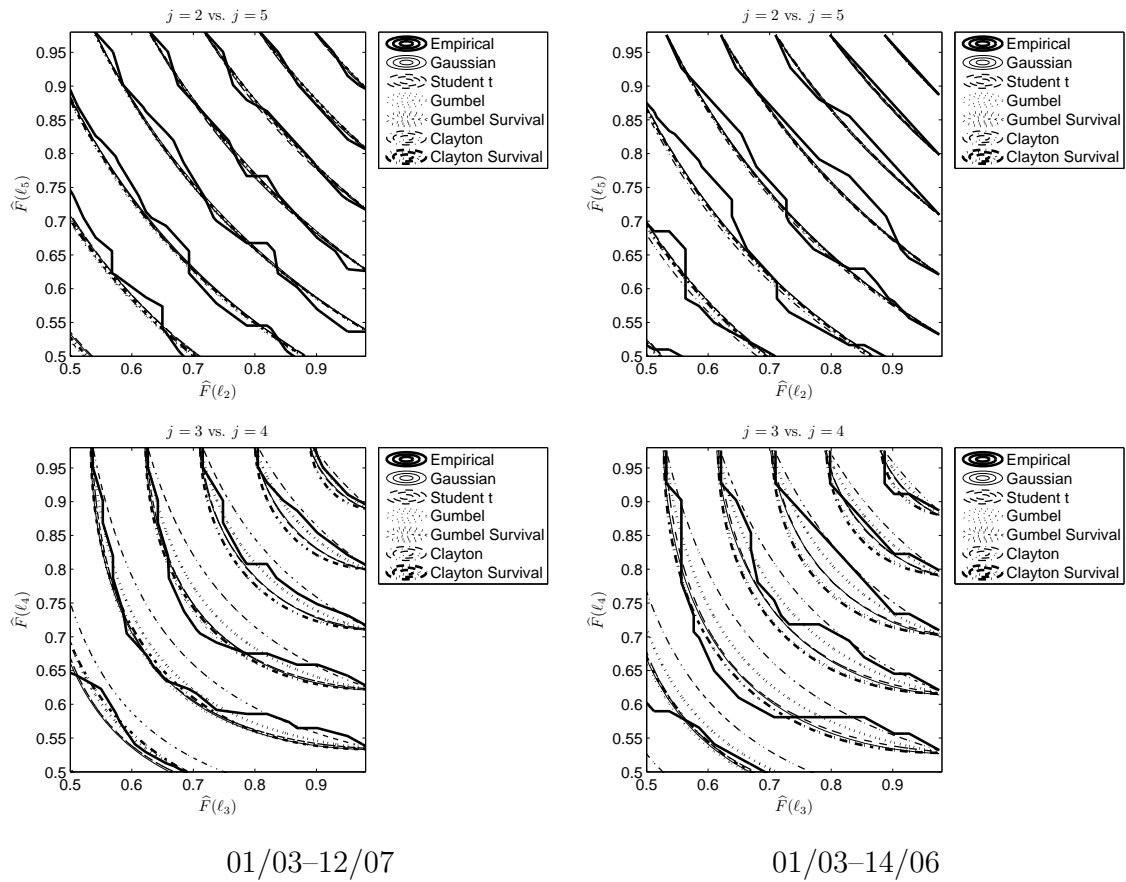


Figure 3: Contour plots of empirical and fitted copulas; Pairs (2;5) (top panel) and (3;4) (bottom panel), full sample (left panel) and partial sample (right panel).

part). This is in line with having no indication of tail dependence for Pair (2;5). The losses in Event-types 3 and 4 do, however, exhibit upper-tail dependence, so that copulas, which are capable of capturing this, fit substantially better.

The challenges of parametric copula estimation in small samples are evident from the kinks in the empirical copulas that are caused by only a few observations. When reducing the sample size, we observe noticeable differences in the empirical copulas, an

indication that also the copula estimates are unstable.

3.4. Nonparametric Tail Dependence

Rather than fitting parametric copulas, we can model tail dependence nonparametrically by replacing the parametric copula in (5) by its empirical counterpart. Here, alternative strategies can be considered. As pointed out in Coles et al. (1999), tail dependence not only implies that $\hat{\lambda}_U > 0$, but also that

$$\bar{\chi} = \lim_{t \rightarrow 1^-} \frac{2 \ln(1-t)}{\ln(\tilde{C}(t,t))} - 1 = 1. \quad (6)$$

where $\tilde{C}(t,t) = \Pr[U_1 > t, U_2 > t]$. As summarized in Table 10, both of these properties can be used to identify the joint tail behavior.

Table 10: Identification of tail behavior.

	Tail Independence	Tail Dependence
λ_U	0	(0,1]
$\bar{\chi}$	[-1,1]	1

For $\lambda_U = 0$, there is asymptotic tail independence, irrespective of the value $\bar{\chi}$ assumes. Asymptotic dependence, however, requires both $\lambda_U > 0$ and $\bar{\chi} = 1$. If asymptotic tail independence holds, the value of $\bar{\chi}$ can still be informative. Rather than taking, in (6), the limit as $t \rightarrow 1$, lower confidence levels may be considered. Dependencies at levels $t < 1$ —a property we refer to as *quantile dependence*—could be present, even if there is no asymptotic tail dependence. Since VaR-estimates are quantile-estimates, quantile dependence is especially relevant for risk-management applications. Given that more observations are available for less extreme confidence levels, nonparametric estimation of quantile dependence via λ_U and $\bar{\chi}$ could—from an empirical viewpoint—be a more reliable option.

Rewriting (5) as

$$\lambda_U = \lim_{t \rightarrow 1^-} \left(2 - \frac{1 - C(t, t)}{1 - t} \right) = \lim_{t \rightarrow 1^-} \lambda_U(t) , \quad (7)$$

quantity $\lambda_U(t)$ conveys information about quantile dependence (see Joe et al., 1992). Alternatively, (7) can be expressed as

$$\lambda_U = \lim_{t \rightarrow 1^-} \left(2 - \frac{\ln(C(t, t))}{\ln(t)} \right) = \lim_{t \rightarrow 1^-} \chi(t) , \quad (8)$$

giving rise to a second quantity, $\chi(t)$, which can be evaluated for different levels of t (Coles et al., 1999). Finally, in (6), $\bar{\chi}$ can be written as

$$\bar{\chi} = \lim_{t \rightarrow 1^-} \frac{2 \ln(1 - t)}{\ln(\tilde{C}(t, t))} - 1 = \lim_{t \rightarrow 1^-} \bar{\chi}(t) . \quad (9)$$

When replacing $C(t, t)$ in (7)–(9) by its empirical counterpart, we obtain estimators $\hat{\lambda}_U(t)$, $\hat{\chi}(t)$ and $\hat{\bar{\chi}}(t)$ for different levels of t . The first two quantities indicate the presence of asymptotic tail dependence, if they converge towards a value above zero as t increases. At the same time, however, $\hat{\bar{\chi}}(t)$ needs to converges to one; otherwise, there is tail independence. In case of tail independence, $\hat{\bar{\chi}}(t)$ may still provide an indication of the presence of quantile dependence.

Figure 4 shows the results for the nonparametric estimation of quantities $\hat{\lambda}_U(t)$, $\hat{\chi}(t)$ and $\hat{\bar{\chi}}(t)$. For Combination (2;5) (upper panel), there is no evidence for tail dependence as all three estimates do not converge to a positive level as t increases. Moreover, $\hat{\bar{\chi}}(t)$ is close to zero over the entire range of t , suggesting also the absence of quantile dependence. The results for Combination (3;4) (lower panel) differ. Although $\hat{\lambda}_U(t)$ and $\hat{\chi}(t)$ drop towards zero for $t > 0.75$, $\hat{\bar{\chi}}(t)$ is clearly positive over a wide range of t , indicating the presence of quantile dependence.

Regarding the choice of the level of t , we are faced with the well-known bias–efficiency tradeoff. Only for high t -levels do we obtain information about the *tail* behavior. As we include more observations away from the tail, estimates become more

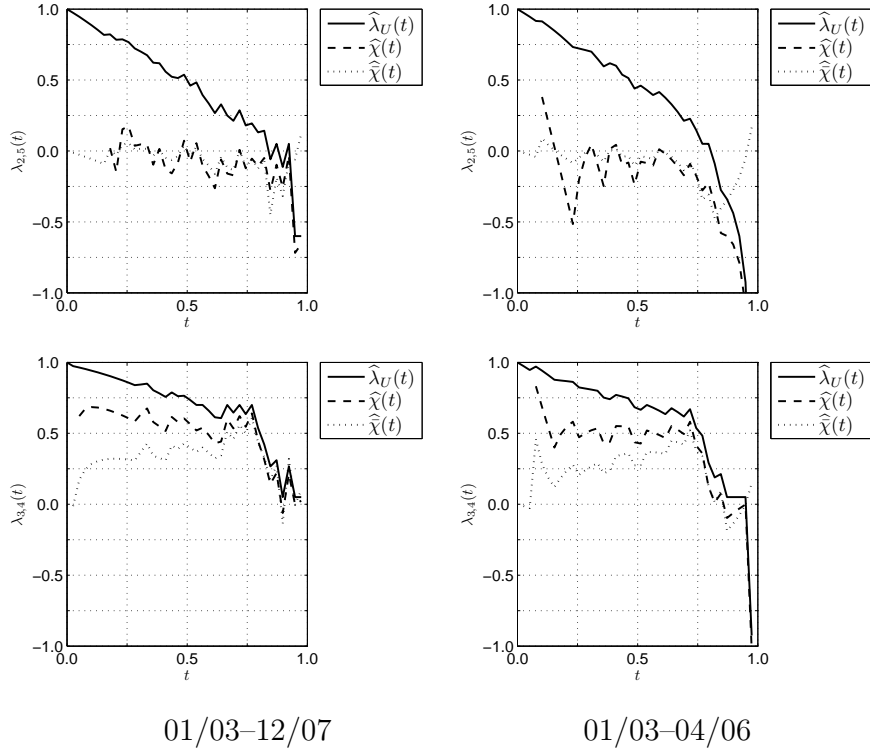


Figure 4: Nonparametric determination of tail dependence for Pairs (2;5) (top panel) and (3;4) (bottom panel), full sample (left panel) and partial sample (right panel).

and more biased. On the other hand, as t approaches unity, tail estimates become increasingly inefficient, given the limited number of observations, so that inference on tail dependence from estimates $\hat{\lambda}_U(t)$ and $\hat{\chi}(t)$ with, say, $t \geq 0.9$ could be questionable. We, therefore, advocate the combined use of alternative approaches to assess dependence properties, rather than relying on one single approach. In our investigation, the results from both parametric copula fitting and the nonparametric analysis suggest tail dependence for Pair (3;4) but not for (2;5).

One lesson from our analysis of two event-type pairs is that we cannot impose a

universal assumption on the joint tail behavior of operational–risk losses across event types. Instead, case–specific empirical analyses, taking the heterogeneity of the event types into account, are required for reliable risk assessment.

4. Effects on Risk Capital

4.1. Range of Risk–capital Estimates

Under the LDA, the aggregate risk–capital estimate for all operational–risk components is obtained from (2) by summing up all individual component–VaRs. This oversimplification should produce only a coarse proxy for an institution’s appropriate capital charge. The crucial question is whether or not the resulting estimate provides a reasonable upper bound. Unfortunately, there is no guarantee for this to be the case. If operational losses are characterized by superadditivity, the standard LDA will induce a downward bias in risk–capital estimates.

To address this question, we simulate $\text{VaR}_{.999}$ –values for the event–type Pairs (2;5) and (3;4), making different dependency assumptions and varying the number of Monte Carlo replications. For all simulations, we use the copula–parameter values obtained by maximum likelihood estimation. To avoid distortions resulting from alternative assumptions about the marginal distributions, lognormal distributions are fitted to the loss data and used to simulate the marginal distributions.

As discussed in Section 2, quantity

$$D_{.999}^{i,j} = \frac{\text{VaR}_{.999}(L_i + L_j) - [\text{VaR}_{.999}(L_i) + \text{VaR}_{.999}(L_j)]}{\text{VaR}_{.999}(L_i) + \text{VaR}_{.999}(L_j)}$$

evaluates the relative difference in risk capital when accounting for dependencies as opposed to naive summation. A positive (negative) value of $D_{.999}^{i,j}$ indicates that the risk–capital estimate from the explicit approach increases (decreases) relative to Basel’s LDA.

In the simulations, we use two extreme dependence assumptions as alternatives to LDA’s comonotonicity assumption, namely, the Gaussian copula and, as a “worst–case scenario,” the copula—among the ones considered—yielding the highest tail dependence coefficient for the respective event–type combination.

Figure 5 shows the results for the Gaussian copula. The differences between the two event–type combinations are evident. Considering, first, the results for 10,000 replications, we find for Combination (2;5), which is characterized by a negative correlation estimate, that risk–capital estimates always decrease; and a risk–capital reduction of almost 30% can be possible. However, for Combination (3;4), with a correlation estimate of about $\hat{\rho} \approx 0.7$, the Gaussian–copula assumption may easily lead to an increase in the risk–capital. Although the interquartile range is entirely below zero, so that a decrease is to be expected in most cases, the estimates from the explicit approach may exceed Basel’s LDA estimates by more than 20%.

As the number of replications increases, the boxplots narrow and the variation of the risk–capital estimates becomes less dramatic. For 50,000 replications, the possible range for Combination (2;5) lies between -19% and -25%; for 100,000, this range becomes somewhat tighter and lies between -19% and -23%. In line with this, for Combination (3;4) we observe that the higher the number of replications, the smaller the portion of cases where risk–capital estimates increase. With 100,000 replications, the maximum increase in risk–capital shrinks to about +3%.

Figure 6 shows the result for the “worst–case copula,” which turns out to be the Student– t (implying a tail–dependence coefficient of $\lambda_U = 0.095$) for Pair (2;5) and the Clayton survival ($\lambda_U = 0.629$) for (3;4). Compared to the Gaussian case in Figure 5, the “worst–case” assumption results in an upward shift of all boxplots. Hence, the presence of tail dependence leads to more cases where risk–capital increases. In case of low tail dependence, as in Pair (2;5), risk–capital still decreases, however, for all

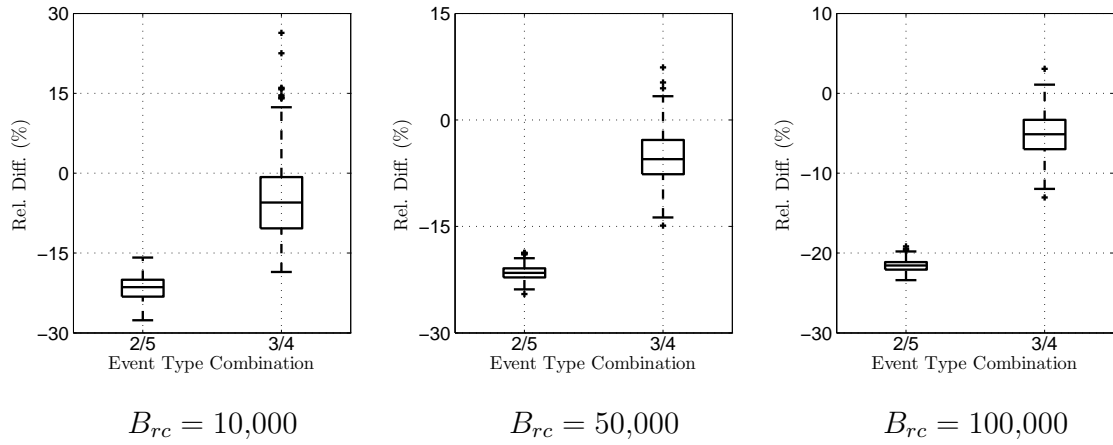


Figure 5: Range of simulated risk capital changes with Gaussian copula.

replications. Higher tail dependence, as in Pair (3;4), may increase risk-capital by up to 6% (100,000 replications).

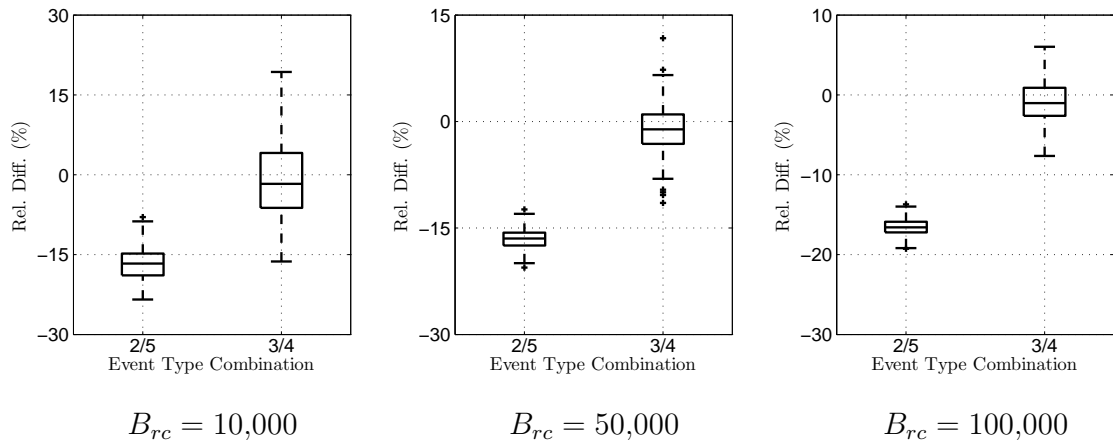


Figure 6: Range of simulated risk capital changes with “worst-case copula.”

The observed increases in risk-capital estimates, when modeling dependencies explicitly rather than summing up, may have two sources: (i) superadditivity affecting

the aggregate VaR measure, and (ii) the number of replications specified in the Monte Carlo setup. In the next subsection we try to disentangle these two effects.

4.2. Bounds on Risk-capital Estimates

As just discussed, it may not be clear to what extent variations in simulation-based assessments of aggregate risk-capital are due to subadditivity problems or to the particular simulation setup. To address this question, we derive a theoretical worst-case bound, so that any exceedances of that bound are due to the simulation setup.¹¹

Copula are always bounded by the so-called Fréchet–Höfding bounds (Fréchet, 1951; Höfding, 1940)

$$\underbrace{\max(u_1 + \dots + u_n - n + 1, 0)}_{C_\ell(\mathbf{u})} \leq C(\mathbf{u}) \leq \underbrace{\min(\mathbf{u})}_{C_u(\mathbf{u})}. \quad (10)$$

By imposing restrictions on the underlying copula, tighter bounds can be established. Given copula C , with lower bound $C_0 \leq C$, the upper bound refers to its dual, $C^d(u_1, u_2) = u_1 + u_2 - C(u_1, u_2)$. Assuming $C_0 \leq C$ and $C^d \leq C_1^d$, we obtain the bounds

$$F_{\min}^{-1}(\alpha) = \inf_{C_0(u_1, \dots, u_n) = \alpha} (F_1^{-1}(u_1) + \dots + F_n^{-1}(u_n)), \quad (11a)$$

$$F_{\max}^{-1}(\alpha) = \sup_{C_1^d(u_1, \dots, u_n) = \alpha} (F_1^{-1}(u_1) + \dots + F_n^{-1}(u_n)). \quad (11b)$$

For the two-dimensional case, $C \geq C_0$ implies that $C^d \leq C_0^d$ and we can take $C_0 = C_1$. The more restrictive the assumptions on the copula bounds are, the tighter the VaR-bounds. In our analysis, we consider the three, increasingly restrictive, cases:

1. $C_0 = C_1 = C_\ell$: We do not use any restriction on the dependence structure and thus use the lower Fréchet bound, C_ℓ ;
2. $C_0 = C_1 = u_i u_j$: We assume that $C \geq u_i u_j$; i.e., we have positive quadrant dependence (PQD);

¹¹The issue of VaR-bounds has, for example, been addressed in Makarov (1981), Frank et al. (1987), and, more recently, in Embrechts and Puccetti (2006).

3. $C_0 = C_{\widehat{CS}}^{\widehat{\gamma}_S}, \bar{C}_1 = C_{\widehat{C}}^{\widehat{\gamma}_C}$: We take the Clayton survival copula as lower bound, using the parameter values estimated from the DIPO data. For the upper bound in (11b), we exploit the fact that $C_0 = C_1$, implying that the survival copula of C_1 must be the Clayton. We, thus, use the parameter estimate $\widehat{\gamma}$ as the upper bound.

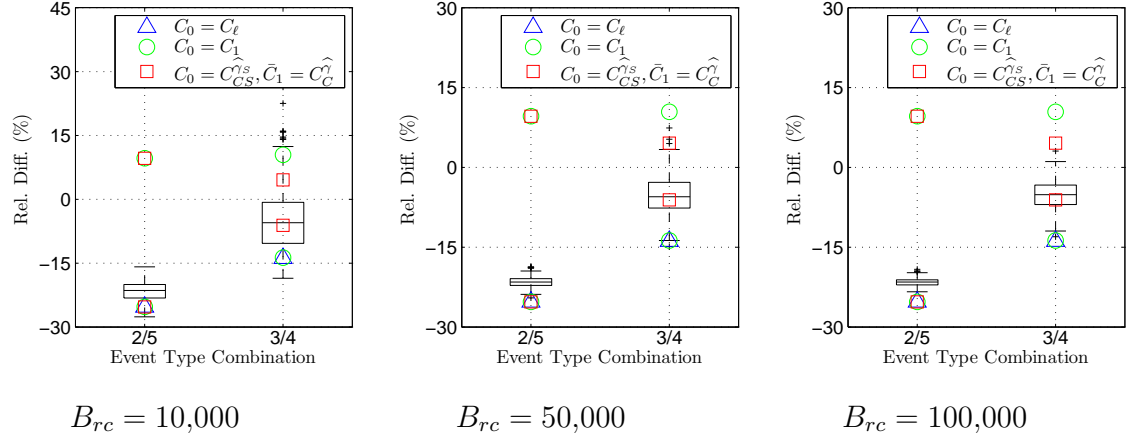


Figure 7: Relative variations in simulated risk capital and theoretical bounds based on a Gaussian copula.

Figures 7 and 8 show again the boxplots presented in Figures 5 and 6, respectively, but now together with the theoretical bounds for the changes in risk-capital under the three assumptions on the copula dependence structures reported above. The boxplots in the left graph in Figures 7 and 8 are for Combinations (2;5) and (3;4) for $B_{rc}=10,000$. The theoretical bounds are indicated by a triangle when $C_0 = C_1 = C_\ell$ (Case 1), a circle when $C_0 = C_1 = u_i u_j$ (Case 2) and square when $C_0 = C_{\widehat{CS}}^{\widehat{\gamma}_S}, \bar{C}_1 = C_{\widehat{C}}^{\widehat{\gamma}_C}$ (Case 3). The center and right graphs in Figures 7 and 8 plot the boxplots for $B_{rc}=50,000$ and $B_{rc}=100,000$, respectively.

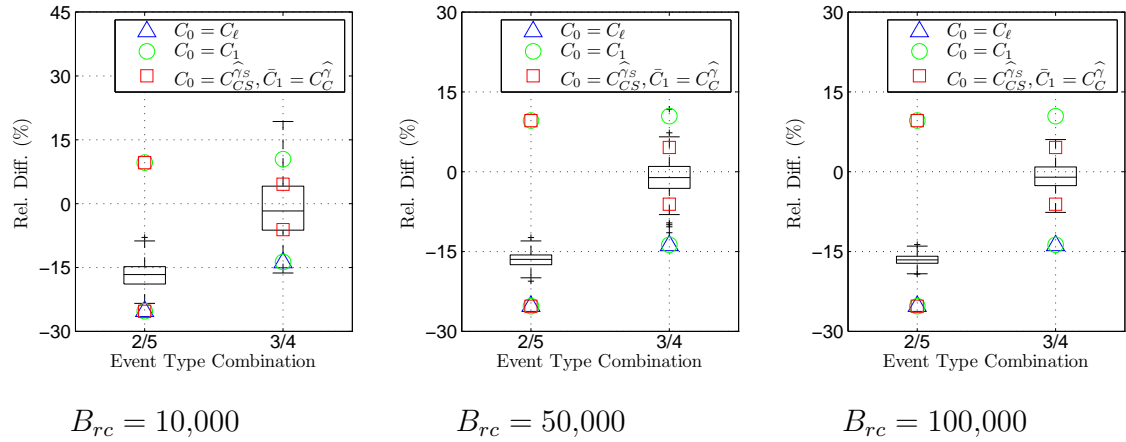


Figure 8: Relative variations in simulated risk capital and theoretical bounds based on a “worst–case copula.”

For analyses with smaller numbers of replications, the bounds in correspondence of no restriction on the dependence can be used to determine whether changes in risk–capital are due to distributional properties of the data or due to the Monte-Carlo setup. In fact, as the figures show, for 10,000 replications, a large part of the boxplots exceeds such theoretical bounds. Hence, the number of simulations is insufficient to get reliable estimates of risk-capital, given that the theoretical bounds for no restriction on the dependence should never be exceeded. As expected, when the number of replications increases, larger portions of the estimates lie inside such bounds until all the estimates are within them.

The use of such bounds can become even more effective to compute risk capital estimates the stronger and more realistic the dependence assumption is. In fact, the bounds get tighter as assumptions on dependence get stronger. Hence, the uncertainty related to the domain of variation of the risk capital estimates decreases and risk capital estimates become more accurate. On the contrary, when no information on the dependence structure is available, if only the bounds computed in correspondence of the dependence assumptions are exceeded by some of the capital estimates, we cannot

determine if the Monte–Carlo setup is adequate or not.

Finally, we also notice that for none of the two event–type pairs are the bounds entirely above or below zero, so that we cannot rule out either an increase or a decrease in aggregate risk–capital estimates. The superadditivity problem is therefore still an issue, even when the Monte–Carlo setup is adequate, and aggregate risk–capital estimates may still increase rather than decrease when modeling dependencies.

Summing up, considering bounds on risk–capital estimates can provide relevant information in order to determine if the number of simulations is sufficient to get valid risk–capital estimates and therefore disentangle the superadditivity effect from the simulation setup. Moreover, the more information on the dependence structure is provided, the smaller the dispersion of the estimates, reducing the uncertainty in the estimates, and possibly, providing more realistic risk–capital estimates that can adequately cover future operational risk losses.

5. Conclusions

Drawing on a real–world database of operational–loss events in the Italian banking industry, we examined different strategies to modeling operational–risk dependencies and assessed their effects on aggregate risk–capital estimates based on 99.9%–VaR. The focus was on two different event–type combinations, representing typical situations when modeling dependence among operational–risk components. One combination was characterized by a lack of (or presence of only weak) dependence, the other by extremal (i.e., tail or quantile) dependence. Despite concentrating only on two cases, it is to be expected that our results will resemble those for other combinations of event types and business lines.

With respect to dependencies among different event types, we confirmed the widely observed phenomenon that linear correlations are unstable and mainly driven by a few extreme loss–events. Rank correlations offer an alternative, but they do not necessarily

display more stability in capturing comovements in monthly loss data. To go beyond the concept of correlation, alternative methods to modeling extremal dependence were considered. Specifically, we employed parametric copula and nonparametric approaches.

Next, we evaluated implications on the estimation of aggregate risk–capital, when modeling dependencies explicitly rather than using Basel’s Loss Distribution Approach, which simply adds up the estimates for the individual risk components. We were interested in the question of whether an explicit modeling of dependencies will—as has been frequently suggested—result in lower aggregate risk–capital estimates. Or can tail dependence lead to larger estimates than the LDA? As our results demonstrate, in the presence of tail dependence, this can, indeed, be the case for Gaussian–copula structures. Therefore, identifying the presence of extremal dependence is of paramount importance when aggregating VaR–based risk estimates. Neglecting tail dependence and restricting one’s view to linear correlation may lead to a substantial underestimation of risk and, thus, may have serious practical implications for financial institutions.

In addition to the purely distributional aspects, another important finding of this study is that for simulation–based VaR–assessment, as is common in practice, the specified number of simulation runs can strongly affect risk–capital calculations. It has to be made sure that the number of runs is sufficiently large to avoid a—potentially severe—overestimation of risk. The increases in aggregate VaR we observed have two sources: the VaR–measure’s lack of subadditivity and the simulation setup. To disentangle the two effects, we considered theoretical, worst–case VaR bounds as a way of controlling the large variation of the estimates and improving the stability of the estimates.

Finally, it should be kept in mind that the determination of operational–risk capital is typically hampered by the limited sample sizes encountered in practice and the explicit focus on extremely small tail probabilities, which are associated with “once–in–thousand–years–events.” Hence, results from either nonparametric or parametric–copula approaches have to be taken with appropriate caution. Nevertheless, two general recommendations follow from our analysis. First, serious efforts should be undertaken

towards improving the empirical database for operational–risk losses; and, second, a combination of modeling strategies should be employed in order to obtain a more complete and reliable picture of dependence structures and their consequences for aggregate risk–capital calculations.

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